7. [8 points] The *Lambda* app team also needs to write some new code and so wants to hire one or more temporary programmers. They will pay \$300 per day to hire an experienced programmer for x days, and \$100 per day to hire a beginner programmer for y days. They want to spend \$900 in total, so

$$9 = 3x + y.$$

However, the team expects that the programmer(s) will drink of a lot of coffee. In particular, they believe the cost C, in dollars, of the coffee they will need can be modeled by

$$C = y - x^3 + 3x^2 + 3x - 8.$$

What value(s) of x will minimize the cost of the coffee? Use calculus to find and justify your answers, and be sure to show enough evidence that the value(s) you find do in fact minimize the cost of the coffee.

Solution: First, we solve for $y = 9 - 3x$ and substitute this into C to get

$$C(x) = (9 - 3x) - x^3 + 3x^2 + 3x - 8 = 1 + 3x^2 - x^3.$$

Then we find all critical points of $C(x)$ using

$$C'(x) = 6x - 3x^2 = 3x(2 - x)$$

which is never undefined and zero at 0 and 2, so our critical points are 0 and 2. We should make sure they are both relevant to our domain: in this case $x \geq 0$ and $y = 9 - 3x \geq 0$, so $x \leq 3$, so our domain is $[0, 3]$ and both critical points are relevant. Now, we find the value of $C(x)$ at both endpoints and the remaining critical point:

x	$C(x)$
0	1
2	5
3	1

We see that both 0 and 3 lead to the minimum cost.

Answer: value(s) of x that minimize the cost of coffee: 0, 3

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