8. [7 points] The function \( f(x) \) is defined as follows:

\[
f(x) = \begin{cases} 
  x & x \leq 0 \\ 
  \frac{x}{x^2 + 1} & x > 0.
\end{cases}
\]

Note that the formula for \( f(x) \) for \( x > 0 \) is unknown. However, it is known that \( f(x) \) is differentiable at each point in its domain \((-\infty, \infty)\), and that \( f'(x) > 0 \) for all \( x \geq 0 \).

a. [4 points] Find the \( x \)-coordinates of all global minimum(s) and global maximum(s) of \( f(x) \) on the interval \((-\infty, 0]\). If there are none of a particular type, write NONE. Use calculus to find your answers, and make sure that you show enough evidence to justify your conclusions.

**Solution:** First, using the quotient rule, we find that \( f'(x) = \frac{1 - x^2}{(x^2 + 1)^2} \) for \( x < 0 \). There are no values of \( x \) where this is undefined, so to find critical points, we set \( f'(x) \) equal to 0 to find that \( x = \pm 1 \). However, this formula is only relevant for \( x < 0 \) so our only critical point for \( x < 0 \) is \( x = -1 \).

Then, we see that

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>(-1)</td>
</tr>
<tr>
<td>( \lim_{x \to -\infty} f(x) )</td>
<td>0</td>
</tr>
</tbody>
</table>

Then we have a global min at \( x = -1 \), and a global max at \( x = 0 \).

**Answer:** Global min(s) at \( x = -1 \)

**Answer:** Global max(es) at \( x = 0 \)

b. [3 points] For each question below, circle all correct answers. No justification is needed.

At which of the following value(s) of \( x \) does \( f(x) \) attain a global minimum on the interval \([-2, 2]\)?

\[
\begin{array}{c|c|c|c|c|c}
 x = -2 & x = -1 & x = 0 & x = 1 & x = 2 & \text{NONE OF THESE}
\end{array}
\]

At which of the following value(s) of \( x \) does \( f(x) \) attain a global maximum on the interval \([-2, 2]\)?

\[
\begin{array}{c|c|c|c|c|c}
 x = -2 & x = -1 & x = 0 & x = 1 & \boxed{x = 2} & \text{NONE OF THESE}
\end{array}
\]