8. [7 points] The function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} \dfrac{x}{x^2+1} & x \leq 0 \\ ? & x > 0. \end{cases}$$

Note that the formula for $f(x)$ for $x > 0$ is unknown. However, it is known that $f(x)$ is differentiable at each point in its domain $(-\infty, \infty)$, and that $f'(x) > 0$ for all $x \geq 0$.

a. [4 points] Find the x-coordinates of all global minimum(s) and global maximum(s) of $f(x)$ **on the interval** $(-\infty, 0]$. If there are none of a particular type, write NONE. Use calculus to find your answers, and make sure that you show enough evidence to justify your conclusions.

Solution: First, using the quotient rule, we find that $f'(x) = \dfrac{1-x^2}{(x^2+1)^2}$ for $x < 0$. There are no values of x where this is undefined, so to find critical points, we set $f'(x)$ equal to 0 to find that $x = \pm 1$. However, this formula is only relevant for $x < 0$ so our only critical point for $x < 0$ is $x = -1$.
Then, we see that

x	$f(x)$
0	0
-1	$-\frac{1}{2}$
$\lim\limits_{x\to-\infty} f(x)$	0

Then we have a global min at $x = -1$, and a global max at $x = 0$.

Answer: Global min(s) at $x =$ -1

Answer: Global max(es) at $x =$ 0

b. [3 points] For each question below, circle all correct answers. No justification is needed.

At which of the following value(s) of x does $f(x)$ attain a global minimum **on the interval** $[-2, 2]$?

$x = -2$ [$x = -1$] $x = 0$ $x = 1$ $x = 2$ NONE OF THESE

At which of the following value(s) of x does $f(x)$ attain a global maximum **on the interval** $[-2, 2]$?

$x = -2$ $x = -1$ $x = 0$ $x = 1$ [$x = 2$]   NONE OF THESE