

1. [10 points] Some values of the invertible, differentiable function  $G(t)$  are shown in the table below, along with some values of  $G'(t)$ , the **derivative** of  $G(t)$ .

$t$	0	1	2	3	4	5	6
$G(t)$	0	2	5	7	8	10	11
$G'(t)$	0	5	1	2	1	3	0

For parts **a.** – **d.**, find the **exact** numerical values, or write DNE if the value does not exist. Your answers should not include the letter  $G$ , but you do not need to simply. *Show your work.*

- a. [2 points] Let  $P(t) = t^3G(t)$ . Find  $P'(2)$ .

*Solution:*  $P'(t) = 3t^2G(t) + t^3G'(t)$ , so

$$P'(2) = 3 \cdot 2^2G(2) + 2^3G'(2) = 12 \cdot 5 + 8 \cdot 1 = 68.$$

**Answer:**  $P'(2) = \underline{\hspace{2cm}68\hspace{2cm}}$

- b. [2 points] Let  $A(t) = \frac{G(3t+2)}{2t+1}$ . Find  $A'(1)$ .

*Solution:*  $A'(t) = \frac{3G'(3t+2)(2t+1) - 2G(3t+2)}{(2t+1)^2}$ , so

$$A'(1) = \frac{3G'(5) \cdot 3 - 2G(5)}{3^2} = \frac{27 - 20}{9} = \frac{7}{9}.$$

**Answer:**  $A'(1) = \underline{\hspace{2cm}7/9\hspace{2cm}}$

- c. [2 points] Let  $K(t) = G^{-1}(t)$ . Find  $K'(2)$ .

*Solution:*  $K'(2) = \frac{1}{G'(G^{-1}(2))} = \frac{1}{G'(1)} = \frac{1}{5}$ .

**Answer:**  $K'(2) = \underline{\hspace{2cm}1/5\hspace{2cm}}$

- d. [2 points] Let  $R(t) = \ln(G(t))$ . Find  $R'(5)$ .

*Solution:*  $R'(t) = \frac{1}{G(t)} \cdot G'(t)$ , so  $R'(5) = \frac{1}{G(5)} \cdot G'(5) = \frac{3}{10}$ .

**Answer:**  $R'(5) = \underline{\hspace{2cm}3/10\hspace{2cm}}$

- e. [2 points] Gabby the gopher is furiously digging an underground tunnel. Suppose  $G(t)$  gives the length in meters of Gabby's tunnel  $t$  hours after she started digging at 6am.

Fill in the blank with a number to give a practical interpretation of the fact that  $G'(5) = 3$ .

*Solution:* The interval from 10:55 to 11:05 is ten minutes, which is one-sixth of an hour, so we need to divide  $G'(5)$  by 6.

Gabby's tunnel was about  $\underline{\hspace{1cm}1/2\hspace{1cm}}$  meters longer at 11:05am than it was at 10:55am.