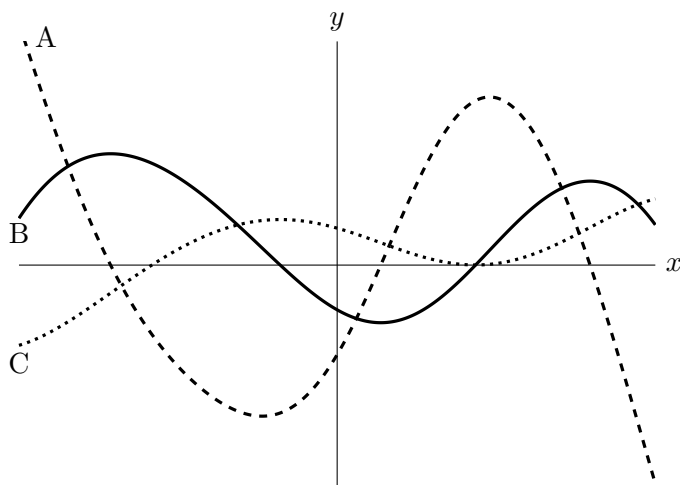


3. [4 points] Shown below are portions of the graphs of the functions  $y = f(x)$ ,  $y = f'(x)$ , and  $y = f''(x)$ . Determine which graph is which, and then, on the answer lines below, indicate after each function the letter A, B, or C that corresponds to its graph. No work or justification is needed.



Answer:  $f(x) : \underline{\text{C}}$

$f'(x) : \underline{\text{B}}$

$f''(x) : \underline{\text{A}}$

4. [8 points] Suppose  $f(x)$  and  $g(x)$  are functions that have exactly the same four critical points, namely at  $x = 1$ ,  $x = 3$ ,  $x = 5$ , and  $x = 7$ . Note that  $f$  and  $g$  have **no other** critical points beyond these four. Assume the first and second derivatives of  $f(x)$  and  $g(x)$  exist everywhere.

The table below shows some values of  $f'(x)$  and  $g''(x)$  at certain inputs. Note that the table gives values of the **first derivative of  $f(x)$**  and the **second derivative of  $g(x)$** .

$x$	0	1	2	3	4	5	6	7	8
$f'(x)$	3	0	-1	0	1	0	2	0	?
$g''(x)$	?	0	-1	-4	?	0	?	2	1

- a. [4 points] Use the table to classify each critical point of  $f$  as a local minimum, maximum, or neither of  $f$ . Circle your answer. If there is not enough information to decide, circle NEI.

- i.  $x = 1$  is a      LOCAL MIN of  $f$       LOCAL MAX of  $f$       NEITHER      NEI
- ii.  $x = 3$  is a      LOCAL MIN of  $f$       LOCAL MAX of  $f$       NEITHER      NEI
- iii.  $x = 5$  is a      LOCAL MIN of  $f$       LOCAL MAX of  $f$       NEITHER      NEI
- iv.  $x = 7$  is a      LOCAL MIN of  $f$       LOCAL MAX of  $f$       NEITHER      NEI

- b. [4 points] Use the table to classify each critical point of  $g$  as a local minimum, maximum, or neither of  $g$ . Circle your answer. If there is not enough information to decide, circle NEI.

- i.  $x = 1$  is a      LOCAL MIN of  $g$       LOCAL MAX of  $g$       NEITHER      NEI
- ii.  $x = 3$  is a      LOCAL MIN of  $g$       LOCAL MAX of  $g$       NEITHER      NEI
- iii.  $x = 5$  is a      LOCAL MIN of  $g$       LOCAL MAX of  $g$       NEITHER      NEI
- iv.  $x = 7$  is a      LOCAL MIN of  $g$       LOCAL MAX of  $g$       NEITHER      NEI

*Solution:* Part **a.** follows from the First Derivative Test, and most of **b.** from the Second Derivative Test. For **b.**(iii.), note that  $g$  must be decreasing on both  $(3, 5)$  and  $(5, 7)$  since  $x = 5$  is the only critical point of  $g$  on  $(3, 7)$  and we have  $g''(3) < 0$  but  $g''(7) > 0$ .