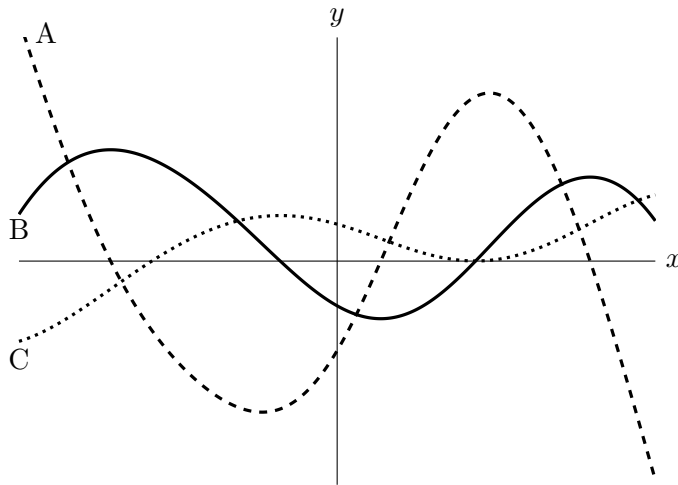


3. [4 points] Shown below are portions of the graphs of the functions $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Determine which graph is which, and then, on the answer lines below, indicate after each function the letter A, B, or C that corresponds to its graph. No work or justification is needed.



Answer: $f(x) : \underline{\text{C}}$
 $f'(x) : \underline{\text{B}}$
 $f''(x) : \underline{\text{A}}$

4. [8 points] Suppose $f(x)$ and $g(x)$ are functions that have exactly the same four critical points, namely at $x = 1$, $x = 3$, $x = 5$, and $x = 7$. Note that f and g have **no other** critical points beyond these four. Assume the first and second derivatives of $f(x)$ and $g(x)$ exist everywhere.

The table below shows some values of $f'(x)$ and $g''(x)$ at certain inputs. Note that the table gives values of the **first derivative of $f(x)$** and the **second derivative of $g(x)$** .

x	0	1	2	3	4	5	6	7	8
$f'(x)$	3	0	-1	0	1	0	2	0	?
$g''(x)$?	0	-1	-4	?	0	?	2	1

- a. [4 points] Use the table to classify each critical point of f as a local minimum, maximum, or neither of f . Circle your answer. If there is not enough information to decide, circle NEI.

- i. $x = 1$ is a LOCAL MIN of f LOCAL MAX of f NEITHER NEI
- ii. $x = 3$ is a LOCAL MIN of f LOCAL MAX of f NEITHER NEI
- iii. $x = 5$ is a LOCAL MIN of f LOCAL MAX of f NEITHER NEI
- iv. $x = 7$ is a LOCAL MIN of f LOCAL MAX of f NEITHER NEI

- b. [4 points] Use the table to classify each critical point of g as a local minimum, maximum, or neither of g . Circle your answer. If there is not enough information to decide, circle NEI.

- i. $x = 1$ is a LOCAL MIN of g LOCAL MAX of g NEITHER NEI
- ii. $x = 3$ is a LOCAL MIN of g LOCAL MAX of g NEITHER NEI
- iii. $x = 5$ is a LOCAL MIN of g LOCAL MAX of g NEITHER NEI
- iv. $x = 7$ is a LOCAL MIN of g LOCAL MAX of g NEITHER NEI

Solution: Part a. follows from the First Derivative Test, and most of b. from the Second Derivative Test. For b.(iii.), note that g must be decreasing on both $(3, 5)$ and $(5, 7)$ since $x = 5$ is the only critical point of g on $(3, 7)$ and we have $g''(3) < 0$ but $g''(7) > 0$.