5. [12 points] A continuous function $h(x)$, its derivative $h'(x)$, and its second derivative $h''(x)$ are given by

$$h(x) = \frac{x}{x^2+1}, \qquad h'(x) = \frac{1-x^2}{(x^2+1)^2}, \qquad \text{and} \qquad h''(x) = \frac{2x(x^2-3)}{(x^2+1)^3}.$$

Note that the critical points of $h(x)$ are ± 1, and the critical points of $h'(x)$ are 0 and $\pm\sqrt{3}$.

For each part below, you must use calculus to find and justify your answers. Be sure your conclusions are clearly stated and that you show enough evidence to support them.

a. [4 points] Find the x-coordinates of all global minima and global maxima of $h(x)$ **on the interval** $[0, 2]$. If there are none of a particular type, write NONE.

Solution: We need to evaluate h at the endpoints of $[0, 2]$ and the critical points of h in $[0, 2]$, and find the greatest and least of these values. The critical points of h are ± 1, so $x = 1$ is the only critical point of h in $[0, 2]$. Since

$$h(0) = 0, \qquad h(1) = \frac{1}{2}, \qquad \text{and} \quad h(2) = \frac{2}{5},$$

on the interval $[0, 2]$ the function h has a global min of 0 at $x = 0$, and a global max of $\frac{1}{2}$ at $x = 1$.

Answer: Global min(s) at $x =$ 0 and Global max(es) at $x =$ 1

b. [4 points] Find the x-coordinates of all global minima and global maxima of $h(x)$ on the interval $(-\infty, \infty)$. If there are none of a particular type, write NONE.

Solution: We need to evaluate h at its critical points $x = \pm 1$, and also find its end behavior as $x \to \pm\infty$. We have $h(1) = \frac{1}{2}$ and $h(-1) = -\frac{1}{2}$, while

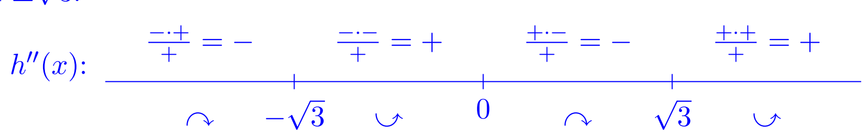
$$\lim_{x\to-\infty} h(x) = 0 = \lim_{x\to\infty} h(x).$$

This means that h has a global min of $-\frac{1}{2}$ at $x = -1$, and a global max of $\frac{1}{2}$ at $x = 1$.

Answer: Global min(s) at $x =$ -1 and Global max(es) at $x =$ 1

c. [4 points] Find the x-coordinates of all inflection points of $h(x)$ on the interval $(-\infty, \infty)$.

Solution: Since h is continuous, its inflection points will occur where its concavity changes, that is, where its second derivative changes sign. So we make a sign chart for $h''(x)$, which is zero at 0 and $\pm\sqrt{3}$:



Because h'' changes sign at each of the points $x = 0, \pm\sqrt{3}$, all three of these points are inflection points of h.

Answer: Inflection point(s) at $x =$ $-\sqrt{3}$, 0, $\sqrt{3}$