(8.) (12 pts) From Exam I, we have that the population of Michigan can be approximated by

$$P = f(t) = 7.8(1.0058)^t,$$

where t is the number of years since the beginning of 1960 and P is in millions.

(a) Determine the average rate of change in the population of Michigan between 1960 and 1980. [Be certain to include units and express your answer as a complete sentence.]

$$\frac{f(20)-f(0)}{20} = \frac{7.8(1.0058)^{20} - 7.8}{20} \approx .04782$$

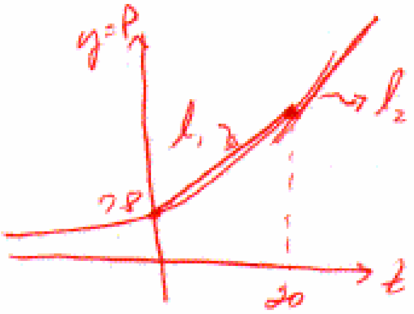
Over the 20-year period from 1960 to 1980, the population increased on average ~ 47,821 people per year.

(b) Determine the (instantaneous) rate of change of the population of Michigan at the beginning of 1980. [Again, use units and a sentence. Show your work.]

The instantaneous rate of change in 1980 is given by $f'(20) = 7.8(1.0058)^{20} \cdot \ln(1.0058) \approx$

Thus, in 1980, the population was increasing at the rate of $\approx .05064$ million people per year -- or ~ 50,640 people per year.

(c) Which is greater—the average rate of change between 1960 and 1980 or the instantaneous change in 1980? Use a graph or tables to give a convincing argument that the rate that you *found* to be greater should indeed *be* greater.



In the figure to the left, the slope of l_1 represents the average rate of change between 1960 & 1980. The slope of l_2 represents the instantaneous change in 1980. Clearly, the slope of l_2 is greater than the slope of l_1. Likewise, the inst. change is greater than the average between 1960 & 1980.

(d) Is there some time, t, such that the instantaneous rate of change of P is equal to the average rate of change from 1960 to 1980? If so, approximate t. If not, explain why not.

Yes, there will be a time between 1960 & 1980 when $f'(t) = \frac{f(20)-f(0)}{20}$. We see this from the graph. Using a calculator to graph $y_1 = 7.8(\ln(1.0058))(1.0058)^x$ + $y_2 = .04782$, we find at $t \approx 10.126$ the slopes are equal.