(9.) (10 pts) Determine a, b, and c so that the graph of the function $f(x) = x^3 + ax^2 + bx + c$ has
a local maximum at $x = -2$,
a local minimum at $x = 1$,
and passes through the point (0,2). [Show your work.]

Given:
$f(x) = x^3 + ax^2 + bx + c$

Since (0,2) is on the graph,
$f(0) = c = 2 \longrightarrow c = 2$

Also, since f is a polynomial, f' is defined for all x. Thus, to have max/min behavior, $f'(-2) = 0$ & $f'(1) = 0$.

Note: $f'(x) = 3x^2 + 2ax + b$

$f'(-2) = 3(4) + 2a(-2) + b = 0$

$\longrightarrow$ ① $12 - 4a + b = 0$

& $f'(1) = 3 + 2a + b = 0 \longrightarrow$ ② $3 + 2a + b = 0$

Subtracting ② from ① gives $9 - 6a = 0$,
so $9 = 6a \rightarrow a = \frac{3}{2}$

Substituting $a = \frac{3}{2}$ into ② gives

$3 + 2\left(\frac{3}{2}\right) + b = 0,$

$3 + 3 + b = 0$

$b = -6$

Thus, $f(x) = x^3 + \frac{3}{2}x^2 - 6x + 2$

$f(0) = 2$ ✓

Note:
$f''(x) = 6x + 3$
$f''(-2) = -12 + 3 = -9 < 0$
$\rightarrow$ loc. max!
$f''(1) = 6 + 3 = 9 > 0$
$\rightarrow$ loc. min!
by 2nd deriv test.