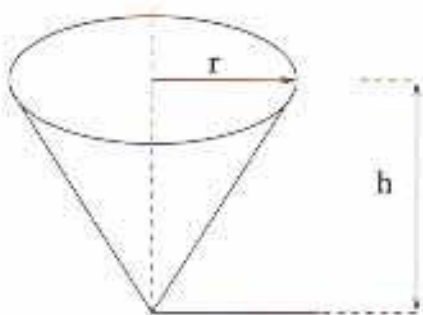


12. (9 points) Fluid flows out of the bottom of a cone-shaped vessel at the rate of 3 cubic cm per second (see figure below). If the radius of the cone is one-third of its height, how fast is the height of the fluid changing when the fluid is 6 cm deep in the center of the cone. Be sure to show your work and give the correct units in your answer. (Remember that the volume of a cone is $\frac{1}{3}\pi r^2 h$.)



Given:

$$V = \frac{1}{3}\pi r^2 h$$

$$\text{and } r = \frac{1}{3}h$$

$$\frac{dV}{dt} = -3 \frac{\text{cm}^3}{\text{sec}}$$

We can write

$$V(h) = \frac{1}{3}\pi \left(\frac{h}{3}\right)^2 \cdot h = \frac{\pi h^3}{9}$$

So

$$\frac{dV}{dt} = \frac{1}{3}\pi \left(\frac{1}{3}h^2\right) \frac{dh}{dt} = \frac{\pi h^2}{9} \frac{dh}{dt}$$

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$$\frac{dh}{dt} \text{ when } h = 6 \text{ cm, using } \frac{dV}{dt} = -3$$

$$-3 = \frac{1}{9}\pi (36) \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{3}{4\pi} \text{ cm/sec}$$