

5. (9 points) Find the equation of the tangent line to the curve $2x^2y^2 - x^3 - y^5 + 1 = 0$ at the point $(2, 1)$.

✓ing point: $8 - 8 - 1 + 1 = 0$

Differentiating:

$$2x^2(2y \frac{dy}{dx}) + 4xy^2 - 3x^2 - 5y^4 \frac{dy}{dx} = 0$$

using $(2, 1)$

$$8(2 \frac{dy}{dx}) + 8 - 12 - 5 \frac{dy}{dx} = 0 \rightarrow y - 1 = \frac{4}{11}(x - 2)$$

$$11 \frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{4}{11}$$

$$y = \frac{4}{11}x - \frac{8}{11} + 1$$

$$y = \frac{4}{11}x + \frac{3}{11}$$

or y-intercept

6. (10 points) (a) Find the Taylor polynomial of degree two that approximates the function $(1 + 2x)^{\frac{3}{2}}$ at $x = 0$ (Show your work!).

$$f(0) = (1)^{\frac{3}{2}} = 1$$

$$f'(x) = \frac{3}{2}(1+2x)^{\frac{1}{2}}(2) = 3(1+2x)^{\frac{1}{2}}$$

$$f'(0) = 3$$

$$f''(x) = \frac{3}{2}(1+2x)^{-\frac{1}{2}}(2)$$

$$f''(0) = 3$$

$$P_2(x) = 1 + 3x + \frac{3}{2}x^2$$

- (b) What is the local linearization of $(1 + 2x)^{\frac{3}{2}}$ near $x = 0$?

$$y = 1 + 3x$$

- (c) Is the local linearization of $(1 + 2x)^{\frac{3}{2}}$ an overestimate or underestimate of the function? Why?

The local linearization is an underestimate because $f''(0) > 0$, so the function is concave up there. In fact, f is concave up for all x , so a linear approximation is an underestimate for all x .

→ (for which the function is defined...)