5. (9 points) Find the equation of the tangent line to the curve $2x^2y^2 - x^3 - y^5 + 1 = 0$ at the point $(2, 1)$.

$\checkmark$ ing point: $8-8-1+1 \stackrel{\checkmark}{=} 0$

Differentiating:

$$2x^2\left(2y\frac{dy}{dx}\right) + 4xy^2 - 3x^2 - 5y^4\frac{dy}{dx} = 0$$

using $(2,1)$

$$8\left(2\frac{dy}{dx}\right) + 8 - 12 - 5\frac{dy}{dx} = 0$$

$$11\frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{4}{11}$$

$$y - 1 = \frac{4}{11}(x-2)$$

$$y = \frac{4}{11}x - \frac{8}{11} + 1$$

$$y = \frac{4}{11}x + \frac{3}{11}$$

or equiv.

6. (10 points) (a) Find the Taylor polynomial of degree two that approximates the function $(1+2x)^{\frac{3}{2}}$ at $x = 0$ (Show your work!).

$f(0) = (1)^{3/2} = 1$

$f'(x) = \frac{3}{2}(1+2x)^{1/2}(2) = 3(1+2x)^{1/2}$

$f'(0) = 3$

$f''(x) = \frac{3}{2}(1+2x)^{-1/2}(2)$

$f''(0) = 3$

$$P_2(x) = 1 + 3x + \frac{3}{2}x^2$$

(b) What is the local linearization of $(1+2x)^{\frac{3}{2}}$ near $x = 0$?

$$y = 1 + 3x$$

(c) Is the local linearization of $(1+2x)^{\frac{3}{2}}$ an overestimate or underestimate of the function? Why?

The local linearization is an underestimate because $f''(0) > 0$, so the function is concave up there. In fact, f is concave up for all x, so a linear approximation is an underestimate for all x... (for which the function is defined...)