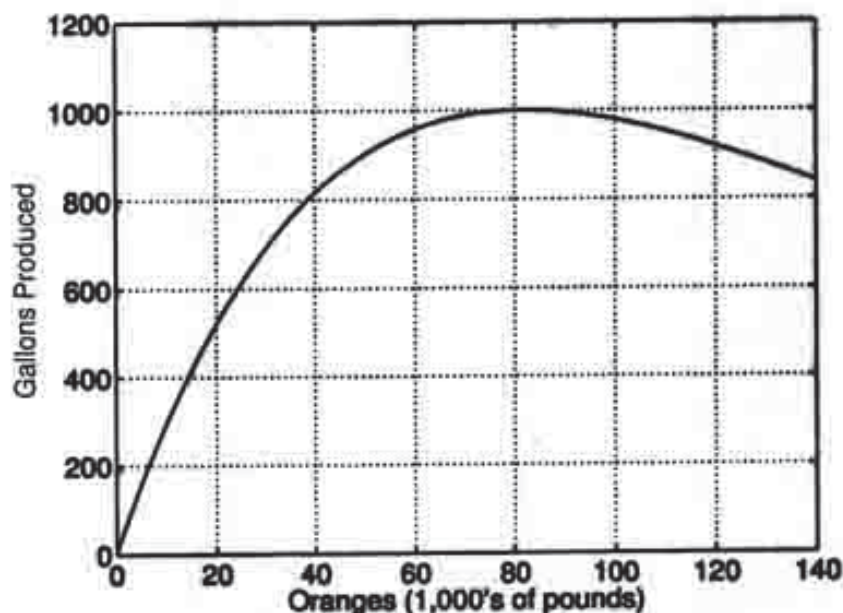


10. (13 points) To qualify as "fresh squeezed," orange juice must be made from oranges which are juiced the same day that they are picked. The manager of the juicing factory attached to the orchard, Mr. I. Squeezem, knows that his factory can handle up to 120,000 pounds of oranges per day. However, due to inefficiencies in storage and complications of the production process when busy, the factory produces only $g(z)$ gallons of orange juice in a day when supplied with z thousand pounds of oranges, where g has the graph given in the figure.



At the orange orchard the harvesting season lasts for 60 days and on day t of the harvesting season

$$f(t) = 80 \sin\left(\frac{\pi t}{60}\right)$$

thousand pounds of oranges are harvested and delivered to the juicing factory.

(a) Mr. Squeezem passed math 115 many years ago. In terms of oranges harvested, explain the meaning of the statement " $f'(20) \approx 2.09$ " to him.

Twenty days into the harvest season, the delivery of oranges to the factory is increasing at the rate of approximately 2,090 lbs per day.

(b) On which day(s) of the harvesting season is the greatest weight of oranges delivered to the factory?

$$f'(t) = \frac{80\pi}{60} \left(\cos \frac{\pi t}{60}\right)$$

day 30

On the interval $(0, 60)$, $f'(t) = 0$ when $t = 30$. Looking at the graph of $f(t)$ on $[0, 60]$, we see that $t = 30$ is a max & since that is the only c.p. it is the global max.

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(c) On which day(s) of the harvesting season are 40,000 pounds of oranges delivered?

From the graph, we see that 40,000 lbs are delivered on days 10 & 50.



(d) From his experience at the factory, Mr. Squeezem knows that when 40,000 pounds of oranges are at the factory, he can produce an additional 10.4 gallons of orange juice from an additional 1000 pounds of oranges. Show Mr. Squeezem how he could express this very simply in terms of the function g if he remembered his calculus.

$$g'(40) = 10.4$$

(e) On the day(s) that 40,000 pounds of oranges are delivered to the factory, at what rate is the number of gallons of orange juice produced changing per day. [Show clearly how you obtain your answer.]

Note that $z = f(t)$, so
 $g(z) = g(f(t))$ and

$$\frac{dg}{dz} = g'(f(t)) \cdot f'(t)$$

Thus, $\frac{dg}{dz} \Big|_{z=40} = g'(40) \cdot \frac{f'(t)}{f'(t)}$
 $\rightarrow z=40$
 $\rightarrow t=10$

Note, when $f(t) = 40$
 $t = 10$ or 50

$$f'(t) = \frac{8\pi}{6} \left(\cos \frac{\pi t}{6} \right)$$

$$f'(10) = \frac{8\pi}{6} \left(\cos \frac{\pi}{6} \right)$$

$$= \frac{4\pi}{3} \left(\frac{\sqrt{3}}{2} \right)$$

$$\approx 3.6276$$

$$f'(50) \approx -3.6276$$

$$\text{Therefore, } \frac{dg}{dz} \Big|_{z=40} = 10.4 (3.6276)$$

$$\approx 37.727$$

The rate is increasing at the rate of approximately
 37.73 gallons/day on day 10
 and decreasing by 37.73 gal/day on day 50.