4. (8 points) A spherical snowball is melting so that its surface area decreases at the constant rate of 40 cm^2 per minute. The surface area and volume of a sphere of radius r are $S = 4\pi r^2$ and $V = 4\pi r^3/3$, respectively. Use this information to answer the following, and remember to include appropriate units in your answers.

(a) How fast is the radius of the snowball changing when the radius is 5 cm?

$S = 4\pi r^2$ given $\frac{dS}{dt} = -40 \frac{cm^2}{min}$

$\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{1}{8\pi r} \cdot \frac{dS}{dt}$

when $r=5$, $\frac{dr}{dt} = \frac{1}{8\pi(5)}(-40) = \frac{-1}{\pi} \frac{cm}{min}$

(b) How fast is the volume changing when the radius is 5 cm?

$V = \frac{4\pi}{3} r^3$

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$, so when $r=5$ $\frac{dV}{dt} = 4\pi(25)\cdot\left(-\frac{1}{\pi}\right)$

$\left.\frac{dV}{dt}\right|_{r=5} = -100\ cm^3/min$

5. (8 points)

(a) Find the tangent line approximation for $f(x) = \frac{x}{x-1}$ near $x = 3$.

near $x=3$, $f(x) \approx f(3) + f'(3)(x-3)$

$= \frac{3}{2} - \frac{1}{4}(x-3)$

$f(3) = \frac{3}{2}$

$f'(x) = \frac{(x-1)-x}{(x-1)^2}$

$= \frac{-1}{(x-1)^2}$

$f'(3) = \frac{-1}{4}$

(b) Is the approximation an overestimate or an underestimate of $f(x)$ for values of x near 3? Explain.

underestimate

We have $f'(x) = -(x-1)^{-2}$,

so $f''(x) = 2(x-1)^{-3}$

$f''(3) > 0$ which indicates f is concave up at $x=3$. In fact, $f''(x) > 0$ for all $x > 1$.

Thus, the approximation near $x=3$ is an underestimate.