

4. (8 points) A spherical snowball is melting so that its surface area decreases at the constant rate of  $40 \text{ cm}^2$  per minute. The surface area and volume of a sphere of radius  $r$  are  $S = 4\pi r^2$  and  $V = 4\pi r^3/3$ , respectively. Use this information to answer the following, and remember to include appropriate units in your answers.

(a) How fast is the radius of the snowball changing when the radius is 5 cm?

$$S = 4\pi r^2$$

$$\text{given } \frac{dS}{dt} = -40 \frac{\text{cm}^2}{\text{min}}$$

$$\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{1}{8\pi r} \cdot \frac{dS}{dt}$$

$$\text{when } r = 5, \quad \frac{dr}{dt} = \frac{1}{8\pi(5)}(-40) = \frac{-1}{\pi} \frac{\text{cm}}{\text{min}}$$

(b) How fast is the volume changing when the radius is 5 cm?

$$V = \frac{4\pi}{3} r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}, \quad \text{so when } r = 5 \quad \frac{dV}{dt} = 4\pi(25) \cdot \left(\frac{-1}{\pi}\right)$$

$$\frac{dV}{dt} \Big|_{r=5} = -100 \text{ cm}^3/\text{min}$$

5. (8 points)

(a) Find the tangent line approximation for  $f(x) = \frac{x}{x-1}$  near  $x = 3$ .

$$\begin{aligned} \text{near } x=3, \quad f(x) &\approx f(3) + f'(3)(x-3) \\ &= \frac{3}{2} - \frac{1}{4}(x-3) \end{aligned}$$

$$f(3) = \frac{3}{2}$$

$$\begin{aligned} f'(x) &= \frac{(x-1) - x}{(x-1)^2} \\ &= \frac{-1}{(x-1)^2} \end{aligned}$$

$$f'(3) = \frac{-1}{4}$$

(b) Is the approximation an overestimate or an underestimate of  $f(x)$  for values of  $x$  near 3?

Explain.

*underestimate*

$$\text{we have } f'(x) = -(x-1)^{-2}$$

$$\text{so } f''(x) = 2(x-1)^{-3}$$

$f''(3) > 0$  which indicates  $f$  is concave up at  $x=3$ . In fact,  $f''(x) > 0$  for all  $x > 1$ . Thus, the approximation near  $x=3$  is an underestimate.