7.(12 points)

(a) Find $\frac{dy}{dx}$ given the equation $y^3 - xy = 2$.

$$3y^2\frac{dy}{dx} - \left(x\frac{dy}{dx} + y\right) = 0$$

$$(3y^2 - x)\frac{dy}{dx} = y$$

$$\frac{dy}{dx} = \frac{y}{3y^2 - x}$$

(b) Is there a point, (x_0, y_0), where the tangent to the curve is horizontal (i.e., parallel to the x-axis)? If so, find one. If not, explain why not.

If $\frac{dy}{dx} = 0$, then $y = 0$. However,
if $y = 0$, then $0 - x(0) = 2 \rightarrow 0 = 2$. *
Thus, there is no solution for $\frac{dy}{dx} = 0$.

(c) Show that the point (3,2) lies on the curve, and find the equation of the tangent line to the curve at (3,2).

$(2)^3 - (3)(2) = 8 - 6 = 2$ ✓ $\therefore$ Thus $(3,2)$

$$\left.\frac{dy}{dx}\right|_{(3,2)} = \frac{2}{12-3} = \frac{2}{9}$$

The line is:

a

or $y = \frac{2}{9}x + \frac{4}{3}$

(d) Use local linearization to find a good approximation for a value of y when the point $(3.09, y)$ lies on the curve. [Show your work.]

$$y \approx 2 + \frac{2}{9}(3.09 - 3) = 2 + \frac{2}{9}(.09)$$
$$= 2 + 2(.01) = 2.02$$