9. (8 points)

(a) Show that the function $f(x) = e^{-x^2/2}$ is concave down on the interval $-1 < x < 1$ and concave up if $x > 1$ or $x < -1$. [Be sure to show your work.]

$$f'(x) = -x\left(e^{-x^2/2}\right)$$

$$f''(x) = (-1)\left(e^{-x^2/2}\right) + (-x)\left(-xe^{-x^2/2}\right)$$

$$= e^{-x^2/2}\left(x^2-1\right) = e^{-x^2/2}(x+1)(x-1)$$

$(x-1)$ - - - - - - - - - 0 + + + +

$(x+1)$ - - - 0 + + + + + +

f'' + -1 − 1 +

Note: $e^{-x^2/2} > 0$ for all x.

$(x-1)(x+1) > 0$ for $x < -1$ or $x > 1 \rightarrow f$ is concave up.

$(x-1)(x+1) < 0$ for $-1 < x < 1 \rightarrow f$ is concave down

(b) Find the member of the family of functions given by $y = e^{-(x-a)^2/b}$ that has a maximum at $x = 3$ and is concave down on the interval $1 < x < 5$.

The function $f(x) = e^{-x^2/2}$ has a max @ $x = 0$. We want the max @ $x = 3$, so we need to shift the graph right by 3 units. Thus, $a = 3$. We also want a horizontal stretch from the max by a factor of 2, so we divide "on the inside" by 2. Thus,

$$y = e^{\frac{-(x-3)^2}{8}}$$