**8.** (10 points) The ideal gas law relates the volume and pressure of a gas to the temperature of the gas. The formula can be given as

$$PV = cT$$

where $P$ is the pressure of the gas measured in *atmospheres*, $V$ is the volume of the gas measured in *liters*, $c$ is a positive constant, and $T$ is the temperature of the gas measured in *kelvins*. (Remember, a temperature measured in kelvins is always positive!)

**(a)** If $T$ is held constant, find $\frac{dV}{dP}$.

Assuming $P \neq 0$ we can rewrite the equation as $V = \frac{cT}{P}$. Now we just use the power rule to obtain $\frac{dV}{dP} = -\frac{cT}{P^2}$.

**(b)** What is the meaning of the sign of your answer to part (a)? Explain this in everyday terms.

Noting that $c, T$, and $P$ are all positive quantities, part (a) tells us that $\frac{dV}{dP}$ is negative. This says that $V$ is a decreasing function of $P$. What this means in everyday terms is that as one increases the pressure on a gas, the volume of the gas goes down.

**(c)** Suppose $V, P$, and $T$ are all functions of the time $t$. Find $\frac{dT}{dt}$.

Taking the derivative with respect to $t$ on each side of the equation we have:

$$V\frac{dP}{dt} + P\frac{dV}{dt} = c\frac{dT}{dt}.$$

Thus, we have

$$\frac{dT}{dt} = \frac{1}{c}\left(V\frac{dP}{dt} + P\frac{dV}{dt}\right).$$