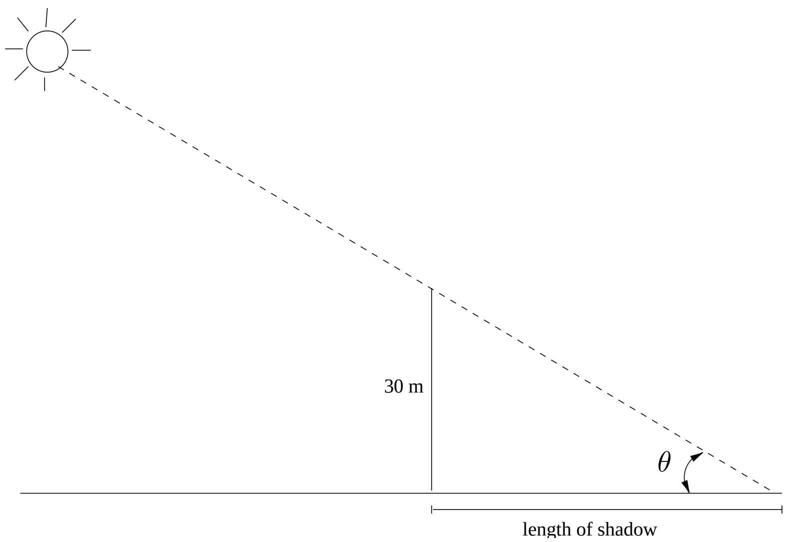
9. (12 points) On a spring day the morning sun is rising at the rate of $\frac{12\pi}{180}$ radians per hour. How fast is the shadow cast by a building that is 30 meters high changing when the sun is $\frac{\pi}{4}$ radians above the horizon in the morning? The following picture may be helpful.



Let x denote the length of the shadow. Then we have the equation $\tan(\theta) = \frac{30}{x}$ relating θ to x. Since we are interested in $\frac{dx}{dt}$, we differentiate this formula with respect to t and obtain:

$$\frac{1}{\cos^2(\theta)} \frac{d\theta}{dt} = -\frac{30}{x^2} \frac{dx}{dt}.$$

Now we are interested in finding $\frac{dx}{dt}$ when $\theta = \frac{\pi}{4}$, so we need to find x, $\frac{1}{\cos^2(\frac{\pi}{4})}$, and $\frac{d\theta}{dt}$ in this case. x can be found from our original equation to be $x = \frac{30}{\tan(\frac{\pi}{4})} = 30$ meters. $\frac{1}{\cos(\frac{\pi}{4})} = \frac{2}{\sqrt{2}}$. So $\frac{1}{\cos^2(\frac{\pi}{4})} = 2$. We are given that $\frac{d\theta}{dt} = \frac{12\pi}{180}$ regardless of the value of θ. So now we just solve for $\frac{dx}{dt}$ and plug in the values we have just found to conclude that

$$\begin{aligned} \frac{dx}{dt} &= -\frac{30^2}{30} \frac{1}{\cos^2(\frac{\pi}{4})} \frac{d\theta}{dt} \\ &= -30 \cdot 2 \cdot \frac{12\pi}{180} \\ &= -12.57 \,\text{m/hr}. \end{aligned}$$

Thus the length of the shadow is decreasing at 12.57 meters per hour when the sun is $\frac{\pi}{4}$ radians above the horizon in the morning.