1. (8 points) The following table gives values of a continuous, differentiable function $f'$ (i.e., the derivative of $f$). The statements below the table concern $f$. For each answer, give the smallest interval that is indicated by the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'(x)$</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>-1</td>
<td>-7</td>
<td>-2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) The function $f$ has a local minimum between $x = \underline{2}$ and $x = \underline{3}$.

(b) The function $f$ has a local maximum between $x = \underline{-1}$ and $x = \underline{0}$.

(c) The function $f$ has an inflection point between $x = \underline{-4}$ and $x = \underline{-2}$. (There is more than one possible answer here.) (There is another between $x=0$ and $x=2$.)

2. (10 points) Let $g$ be a function such that $g(2) = 4$ and whose derivative is known to be $g'(x) = \sqrt{x^2 + 2}$.

(a) Use a linear approximation to estimate the value of $g(1.95)$. Show your work.

We know that $g(2 + \Delta x) \approx g'(2)(\Delta x) + g(2)$. Since we’re looking for $g(1.95)$, we set $\Delta x = -0.05$. Also, $g'(2) = \sqrt{2^2 + 2} = \sqrt{6}$. Plugging these values into the above formula gives $g(1.95) \approx \sqrt{6}(-0.05) + 4 \approx 3.878$.

(b) Do you think your estimate in part (a) is an overestimate or an underestimate? Explain.

We first calculate $g''(x) = \frac{1}{2}(2x)(x^2 + 2)^{-\frac{1}{2}} = \frac{2x}{\sqrt{x^2 + 2}}$. So, $g''$ is positive for all $x > 0$. Thus, the estimate in part (a) is an underestimate since the tangent line to the graph of $g$ at $x = 2$ lies below the actual graph of $g$. 