

1. (8 points) The following table gives values of a continuous, differentiable function f' (i.e., the derivative of f). The statements below the table concern f . For each answer, give the smallest interval that is indicated by the table.

x	-4	-3	-2	-1	0	1	2	3	4
$f'(x)$	3	4	3	2	-1	-7	-2	4	6

(a) The function f has a local minimum between $x = \underline{2}$ and $x = \underline{3}$.

(b) The function f has a local maximum between $x = \underline{-1}$ and $x = \underline{0}$.

(c) The function f has an inflection point between $x = \underline{-4}$ and $x = \underline{-2}$. (There is more than one possible answer here.) (There is another between $x=0$ and $x=2$.)

2. (10 points) Let g be a function such that $g(2) = 4$ and whose derivative is known to be $g'(x) = \sqrt{x^2 + 2}$.

(a) Use a linear approximation to estimate the value of $g(1.95)$. Show your work.

We know that $g(2 + \Delta x) \approx g'(2)(\Delta x) + g(2)$. Since we're looking for $g(1.95)$, we set $\Delta x = -0.05$. Also, $g'(2) = \sqrt{2^2 + 2} = \sqrt{6}$. Plugging these values into the above formula gives $g(1.95) \approx \sqrt{6}(-0.05) + 4 \approx 3.878$.

(b) Do you think your estimate in part (a) is an overestimate or an underestimate? Explain.

We first calculate $g''(x) = \frac{1}{2}(2x)(x^2 + 2)^{-\frac{1}{2}} = \frac{x}{\sqrt{x^2 + 2}}$. So, g'' is positive for all $x > 0$. Thus, the estimate in part (a) is an underestimate since the tangent line to the graph of g at $x = 2$ lies below the actual graph of g .