3. (16 points) Some values for a differentiable function $f$ are given in the table below, and the graph of $y=g(x)$ on the interval $[-6,7]$ is given in the figure below. Do not assume any information about $f$ or $g$ other than what is given.


| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 0.5 | 4 | 7.5 | 10 | 9 | 5 | 0 | 3 | 9 |
| $f^{\prime}(x)$ | 3 | 4 | 3 | 2 | -1 | -7 | -2 | 4 | 6 |

Use the table and the graph to find the following, if possible. If any information is missing, explain clearly what is missing. Show your work.
(a) Find $h^{\prime}(4)$ if $h(x)=g(x) f(x)$.

We know $h^{\prime}(x)=g^{\prime}(x) f(x)+g(x) f^{\prime}(x)$. So, $h^{\prime}(4)=g^{\prime}(4) f(4)+g(4) f^{\prime}(4)$. From the table, we can see that $f(4)=9$ and $f^{\prime}(4)=6$. From the graph of $g$, we can see that $g(4)=-1$. Computing the slope of the line at $g=4$ gives $g^{\prime}(4)=-2$. Plugging these values into the above formula for $h^{\prime}(4)$ gives that $h^{\prime}(4)=(-2)(9)+(-1)(6)=-24$.
(b) Find $h^{\prime}(4)$ if $h(x)=g(f(x))$.

We know $h^{\prime}(x)=g^{\prime}(f(x)) f^{\prime}(x)$. So, $h^{\prime}(4)=g^{\prime}(f(4)) f^{\prime}(4)$. From the table, we can see that $f(4)=9$ and $f^{\prime}(4)=6$. However, we can not determine $g^{\prime}(9)$ from the given graph of $g$. So there is MISSING INFORMATION $-g^{\prime}(9)$.
(c) Find $h^{\prime}(-2)$ if $h(x)=4 \sin (g(x))-\pi$.

We know that $h^{\prime}(x)=4 \cos (g(x)) g^{\prime}(x)$. So, $h^{\prime}(-2)=4 \cos (g(-2)) g^{\prime}(-2)$. From the graph of $g, g(-2)=0$ and $g^{\prime}(-2)=\frac{-2}{3}$ (compute the slope of the line through $x=-2$ ). Plugging in these values into the above equation gives $h^{\prime}(-2)=4(-2 / 3)=-8 / 3$.
(d) Find $h^{\prime}(1)$ if $h(x)=(g(x))^{2}$.

We know that $h^{\prime}(x)=2 g(x) g^{\prime}(x)$. So, $h^{\prime}(1)=2 g(1) g^{\prime}(1)$. Notice though that on the graph of $g$, there is a sharp point at $x=1$. This means that $g^{\prime}(1)$ is undefined, and thus finding $h^{\prime}(1)$ is NOT POSSIBLE.

