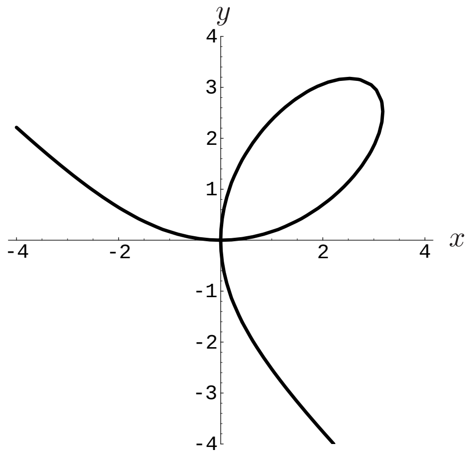
4. (12 points) An example of Descartes' folium, shown in the picture below, is given by $x^3 + y^3 = 6xy$.



(a) Show that the point (3,3) is on the graph.

To show that the point (3,3) is on the graph, we must check that plugging in $x = 3$ and $y = 3$ into the given equation makes both the left hand side and the right hand side equal to each other. Indeed, $x^3 + y^3 = 3^3 + 3^3 = 54$ and $6xy = 6(3)(3) = 54$, so the point (3,3) is on the graph.

(b) Find the equation of the tangent to the graph at the point $(3, 3)$. Show your work.

We must first implicitly differentiate the given equation:
$3x^2 + 3y^2\frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$
$3y^2\frac{dy}{dx} - 6x\frac{dy}{dx} = 6y - 3x^2$
$\frac{dy}{dx}(3y^2 - 6x) = 6y - 3x^2$
$\frac{dy}{dx} = \frac{6y-3x^2}{3y^2-6x}$.
So, at the point (3,3), the slope of the tangent to the curve equals $\frac{6(3)-3(3^2)}{3(3^2)-6(3)} = \frac{18-27}{27-18} = -1$. The equation of the tangent line is then $y - 3 = -(x - 3)$ or $y = -x + 6$.

(c) For what value(s) of x (if any) will the tangent to this curve be horizontal? [You do not need to solve for both x and y–just show x in terms of y.] Show your work.

A tangent to the curve will be horizontal if $\frac{dy}{dx} = 0$. This will only happen if the numerator in the equation found for $\frac{dy}{dx}$ in part (b) equals zero. So, we set $6y - 3x^2 = 0$ and solve for x. This gives:
$3(2y - x^2) = 0$
$2y - x^2 = 0$
$2y = x^2$. So, $x = \pm\sqrt{2y}$.