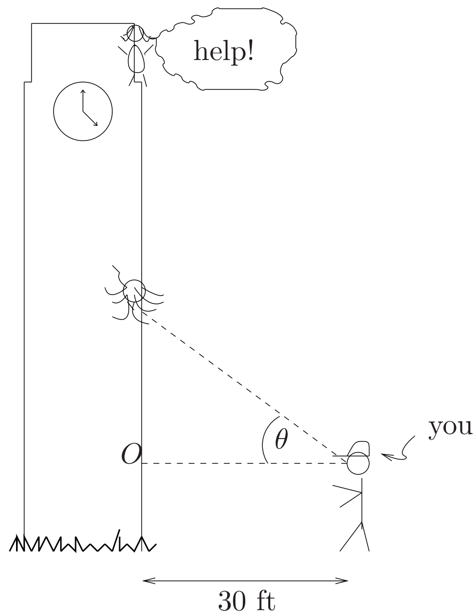
6. (10 points) Dr. Octopus is holding Mary Jane Parker hostage at the top of Burton Tower. Spiderman decides to climb the bell tower to try and rescue Mary Jane. Suppose you are standing 30 ft. away from the base of the tower watching Spiderman as he climbs. Let θ be the angle between the line of your horizon and your line of sight to Spiderman. The picture below may help you. [Picture not to scale.]



(a) Find a formula for the rate of change of Spiderman's distance from the point O with respect to θ.

Let x be Spiderman's distance from the point O. So a formula for the rate of change of Spiderman's distance from the point O will be given by $\frac{dx}{d\theta}$. Notice that $\tan(\theta) = \frac{x}{30}$. So, $\frac{dx}{d\theta} = \frac{30}{\cos^2\theta}$.

(b) If the distance from point O to Mary Jane is 200 ft. and Spiderman is climbing at a constant 8 ft/sec, what is the rate of change of θ with respect to time when Spiderman reaches Mary Jane?

We will use the formula found in (a) to solve this part. We're given that $\frac{dx}{dt} = 8$ ft/sec and we are trying to determine $\frac{d\theta}{dt}$. So we must figure out the value of θ when Spiderman is at the top of the bell tower. We do this by solving for θ in the equation:
$\tan(\theta) = \frac{200}{30}$. So, $\theta = \tan^{-1}(\frac{20}{3})$. Plugging these values into the equation from (a) gives:

$$8 = \frac{30}{(\cos(\tan^{-1}(\frac{20}{3})))^2}\frac{d\theta}{dt}.$$

So, $\frac{d\theta}{dt} \approx 0.0059$ radians/sec.