

8. (20 points) For each of the following, circle *all* correct answers. In each case, there may be more than one item which is correct.

(a) The function  $f'$  is continuous everywhere and changes from negative to positive at  $x = a$ . Which of the following *must* be true?

- $a$  is a critical point of  $f$ .
- $f(a)$  is a local maximum of  $f$ .
- $f(a)$  is a local minimum of  $f$ .
- $f'(a)$  is a local maximum.
- $f'(a)$  is a local minimum.

(b) A function  $g$  is defined on all points of a closed interval. Which of the following *must* be true?

- $g$  must have both a global maximum *and* a global minimum.
- $g$  is differentiable on the interval.
- $g$  has no critical points.
- $g$  is continuous on the interval.
- None of the above statements *must* be true.

(c) For the graph of a cubic polynomial  $ax^3 + bx^2 + cx + d$ , ( $a > 0$ ), the signs of  $f'(0)$ ,  $f''(0)$  and  $f'''(0)$  (respectively) could be which of the following? (Circle all that are possible.)

- $-, 0, +$
- $-, 0, -$
- $+, +, +$
- $-, +, -$
- $+, -, +$

(d) The graph of  $y = h(x)$  has a local max at  $x = 3$  on the closed interval  $[0,5]$ . Which of the following *must* be true?

- $h'(3)$  is equal to zero or  $h(3)$  is an end point.
- $h$  has a critical point at  $x = 3$ .
- $h''(3)$  is positive.
- $h''(3)$  is negative.
- None of the statements *must* be true.

(e) Which of the following *cannot* be computed using L'Hopital's rule?

- $\lim_{x \rightarrow 0} (\sin x / x)$
- $\lim_{x \rightarrow 0} (\cos x / x)$
- $\lim_{x \rightarrow 0} (x / \sin x)$
- $\lim_{x \rightarrow \infty} (x / e^x)$
- $\lim_{x \rightarrow \infty} (\sin x / x)$