1. (20 points) A small company called Maple, Inc. is designing a fancy gift box with a square base. The box must have a volume of 3000 $\text{cm}^3$. The gift box has a lid which is to be made of a material that costs \$1 per square centimeter. The material for the sides of the box costs \$0.75 per $\text{cm}^2$, and the material for the bottom is \$0.80 per $\text{cm}^2$.

   (a) (10 pts.) What are the dimensions of the cheapest gift box the company can make?

   Let $x$ stand for the side of the square base/lid, $h$ stand for the height of the box, and $C$ stand for the cost of making one gift box.

   We then have:

   - $x^2h = 3000$;
   - $(0.80 + 1)x^2 + 4(0.75)xh = C$;

   Solving for $h = 3000/x^2$ in the first equation, and substituting into the second, we obtain the following cost equation:

   $$C = 1.8x^2 + +9000/x.$$

   Solving for $x$ in $C'(x) = (3.6x^3 - 9000)/x^2 = 0$, we obtain $x = 2500^{1/3} \simeq 13.57$ cm. So, for this value of $x$, $h = 3000/2500^{2/3} \simeq 16.29$ cm.

   Since

   $$C''(x) = 3.6 + 18000/x^3 > 0 \text{ for all } x > 0,$$

   we see that:

   - $C''(2500^{1/3}) > 0$, and $C$ has a local minimum at $x = 2500^{1/3}$,
   - This local minimum is a global minimum since $C(x)$ is concave up for $x > 0$.

   So, the **dimensions of the cheapest gift box** are approximately

   $$x \simeq 13.57 \text{ cm and } h \simeq 16.29 \text{ cm},$$

   where $h$ is the height of the box, and $x$ is the length of the side of the square base.

It turns out that Maple, Inc. also produces a cube-shaped wooden box to store jewelry. The cost of producing $q$ of these boxes is given by

$$C(q) = 8600 + 0.0001(q - 80)^3(q + 90).$$

   (b) (3 pts.) What is the marginal cost when 80 boxes are made? Show your work.

   Marginal cost when 80 boxes are made: $C'(80)$

   Since,

   - $C'(q) = 0.0003(q - 80)^2(q + 90) + 0.0001(q - 80)^3$, and
   - $C'(80) = 0$;

   then the marginal cost when 80 boxes are made is *zero dollars per box.*

*(This problem continues on the next page.)*

(*This is a continuation of Problem 1).*

(c) (3 pts.) The marginal cost of producing 95 of the cube-shaped jewelry boxes is about \$13 per box. Explain what this means in practical terms. (Your explanation should be understandable to someone who does not know calculus or economics language).

The cost of producing 96 boxes is about \$13 more than the cost of producing 95 boxes.

(d) (4 pts.) Let $R$ and $P$ denote, respectively, the revenue and the profit of Maple, Inc. from selling $q$ of the cube-shaped jewelry boxes. Fill in the blank and circle the right choice in the paragraph below, as indicated.

If the profit $P$ is maximized when 95 jewelry boxes are sold, then

$R'(95) =$ 13 dollars per box *(fill in the blank)*, and $P''(95)$ must be

POSITIVE / NEGATIVE / ZERO *(circle the appropriate choice).*