2. (10 points) Suppose f has a continuous derivative whose values are given in the following table.

x	0	1	2	3	4	5	6	7	8	9	10
$f'(x)$	5	2	1	-2	-5	-3	-1	2	3	1	-1

(a) Using the data in the table, estimate x-coordinates of indicated critical points of f for $0 < x < 10$.

Since f' is continuous, f' is never undefined and the only critical points are values of x for which $f'(x) = 0$. From the table above, we see that f' changes sign between $x = 2$ and $x = 3$, between $x = 6$ and $x = 7$, and between $x = 9$ and $x = 10$. So, we estimate f has 3 critical points at:

- $x \simeq 2.5$,
- $x \simeq 6.5$,
- $x \simeq 9.5$.

(b) For each critical point above, indicate if it is a local maximum of f, a local minimum, or neither.

- $x \simeq 2.5$ is a local maximum.
  (Since f' changes sign from positive to negative as one moves from left to right in a small neighborhood about $x \simeq 2.5$).
- $x \simeq 6.5$ is a local minimum.
- $x \simeq 9.5$ is a local maximum.

(c) Approximate interval(s) between $x = 0$ and $x = 10$, if any, for which the data indicates that the graph of f is concave up?

The function f is concave up wherever f' is increasing (or where f'' is positive). Looking at the table, we see that f' is increasing approximately for

$$4.5 \leq x \leq 8.5.$$

So, we estimate f is concave up when $4.5 \leq x \leq 8.5$.

(d) If $f(0) = 4$, approximate the value of $f(0.2)$.

The best linear approximation for f at the point (0,4) is given by $y = f'(0)x + b = 5x + b$. Substituting $x = 0$, $y = 4$ in this linear equation, we find $b = 4$.

Therefore,

$$f(x) \simeq 5x + 4 \textbf{ near } x = 0.$$

So, $f(0.2) \simeq 5(0.2) + 4 = 5$, is the approximate value of $f(0.2)$.