4. (6 points) Find the *exact* equation of the linear approximation to the curve $f(x) = 10e^{0.4x}$ having slope equal to 2.

- We want x so that $f'(x) = 4e^{0.4x} = 2$;
- Solving, we find that $x = \ln(0.5)/0.4$;
- Now, $f(\ln(0.5)/0.4) = 5$;
- So, the linear approximation we want is of the form

$$f(x) \simeq 5 + 2(x - \ln(0.5)/0.4).$$

5. (10 points) Find the *exact* coordinates of the point (x, y) where the tangent line to the graph of

$$y^3 - xy = -6$$

is vertical. You should start by differentiating the equation above implicitly with respect to x. Show step-by-step work.

Differentiating implicitly with respect to x we get,

$$\begin{aligned} 3y^2y' - [y + xy'] &= 0 \\ y'(3y^2 - x) - y &= 0 \\ y' &= \frac{y}{3y^2 - x}. \end{aligned}$$

The last expression for y' is undefined if $3y^2 - x = 0$ or $x = 3y^2$. We substitute this expression for x in the original equation to get:

$$\begin{aligned} y^3 - 3y^3 &= -6 \\ y &= 3^{1/3}. \end{aligned}$$

This means that $x = 3y^2 = 3^{5/3}$ when $y = 3^{1/3}$, and so

$$(x, y) = (3^{5/3}, 3^{1/3})$$

are the exact coordinates of the point we want.