- 4. (6 points) Find the exact equation of the linear approximation to the curve  $f(x) = 10e^{0.4x}$  having slope equal to 2.
  - We want x so that  $f'(x) = 4e^{0.4x} = 2$ ;
  - Solving, we find that  $x = \ln(0.5)/0.4$ ;
  - Now,  $f(\ln(0.5)/0.4) = 5$ ;
  - So, the linear approximation we want is of the form

$$f(x) \simeq 5 + 2(x - \ln(0.5)/0.4).$$

5. (10 points) Find the exact coordinates of the point (x, y) where the tangent line to the graph of

$$y^3 - xy = -6$$

is vertical. You should start by differentiating the equation above implicitly with respect to x. Show step-by-step work.

Differentiating implicitly with respect to x we get,

$$3y^{2}y' - [y + xy'] = 0$$
$$y'(3y^{2} - x) - y = 0$$
$$y' = \frac{y}{3y^{2} - x}.$$

The last expression for y' is undefined if  $3y^2 - x = 0$  or  $x = 3y^2$ . We substitute this expression for x in the original equation to get:

$$y^3 - 3y^3 = -6$$
$$y = 3^{1/3}.$$

This means that  $x = 3y^2 = 3^{5/3}$  when  $y = 3^{1/3}$ , and so

$$(x,y) = (3^{5/3}, 3^{1/3})$$

are the exact coordinates of the point we want.