6. (12 points) The functions $r = f(t)$ and $V = g(r)$ give the radius and the volume of a commercial hot air balloon that is being inflated for testing. The variables t and r are measured in minutes and feet respectively, while the volume V is measured in cubic feet. The inflation begins at $t = 0$.

Use the information on the tables below to answer questions (i)-(iii). (Question (iv) is independent of the tables.)

t	$f(t)$	$f'(t)$
0	c	d
30	b	x
60	a	z

r	$g(r)$	$g'(r)$
a	b	x
b	c	z
d	x	y

(i) (2 pts.) How fast is the radius of the balloon increasing initially?

d ______________________ ft/min.

(ii) (2 pts.) Assuming f is always increasing for $0 < t < 60$, how much time has ellapsed (since inflation began) when the radius is growing by z ft/min?

60 ______________________ minutes.

(iii) (3 pts.) How fast is the volume of the balloon increasing a half hour after inflation began?

zx ______________________ ft^3/min.

(iv) (5 pts.) *(This item is independent of the previous ones).* It turns out that the balloon's surface area increases with the radius by the formula

$$S = h(r) = 4\pi r^2$$

If the radius of the balloon increases linearly from 5 feet at a rate of 1.5 feet per minute, how fast is the balloon's surface area growing an hour after inflation began? Show your work.

Note first that,

$$r = f(t) = 1.5t + 5.$$

Then,

$$\begin{aligned} \left.\frac{d}{dt}\right|_{t=60} h(f(t)) &= \left(\left.\frac{dh}{dr}\right|_{r=f(60)}\right)\left(\left.\frac{df}{dt}\right|_{t=60}\right) \\ &= h'(f(60))f'(60) \\ &= h'(95)f'(60) = h'(95)(1.5) \\ &= 8\pi(95)(1.5) = 1140\pi \text{ ft}^2/\text{min}. \end{aligned}$$

So the surface area of the balloon is growing at a rate of 1140π square feet per minute one hour after inflation began.