

5. (10 points) Find the quadratic polynomial $g(x) = ax^2 + bx + c$ which “best fits” the function $f(x) = \ln(x)$ at $x = 1$ in the sense that

$$g(1) = f(1), \quad \text{and} \quad g'(1) = f'(1), \quad \text{and} \quad g''(1) = f''(1).$$

$$f(x) = \ln(x) \Rightarrow f(1) = \ln(1) = 0$$

$$f'(x) = \frac{1}{x} \Rightarrow f'(1) = 1$$

$$f''(x) = -\frac{1}{x^2} \Rightarrow f''(1) = -1$$

$$g(x) = ax^2 + bx + c \Rightarrow g(1) = a + b + c$$

$$g'(x) = 2ax + b \Rightarrow g'(1) = 2a + b$$

$$g''(x) = 2a \Rightarrow g''(1) = 2a$$

Thus, using the given equalities, we find

$$2a = g''(1) = f''(1) = -1 \Rightarrow a = -\frac{1}{2}$$

$$2\left(-\frac{1}{2}\right) + b = g'(1) = f'(1) = 1 \Rightarrow b = 2$$

$$-\frac{1}{2} + 2 + c = g(1) = f(1) = 0 \Rightarrow c = -\frac{3}{2}$$

$$g(x) = \underline{\underline{-\frac{1}{2}x^2 + 2x - \frac{3}{2}}}$$