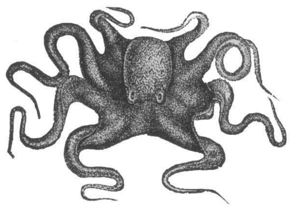
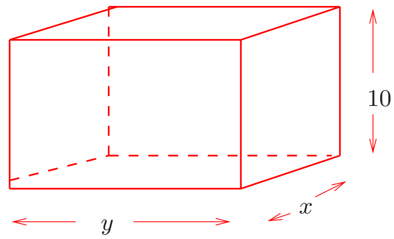
7. (14 points) No matter what is done with the other exhibits, the octopus tank at the zoo must be rebuilt. (The current tank has safety issues, and there are fears that the giant octopus might escape!) The new tank will be 10 feet high and box-shaped. It will have a front made out of glass. The back, floor, and two sides will be made out of concrete, and there will be no top. The tank must contain at least 1000 cubic feet of water. If concrete walls cost \$2 per square foot and glass costs \$10 per square foot, use calculus to find the dimensions and cost of the least-expensive new tank. [Be sure to show all work.]



GIANT OCTOPUS (*Enteroctopus*)[2]



Denote the width of the tank by x and the length of the tank by y. The height of the tank is given as 10 feet. We know that the volume of the tank must be at least 1000 cubic feet, so let V denote the desired volume of the tank, where $V \geq 1000$. Then for a fixed value of V, we know that $10xy = V$, so that x and y are related by the equation $x = \frac{V}{10y}$. Now assuming that one of the y x 10 sides is the front of the tank (i.e, the glass panel), the total cost of the tank is given by:

$$C = 10(10y) + 2[(2)10x + 10y + xy] = 120y + 40x + 2xy.$$

Substituting for x, we can write C as a function of one variable:

$$C(y) = 120y + \frac{4V}{y} + \frac{V}{5}$$

Since the cost function increases as V increases, in order to minimize the cost to build the tank we must have V be as small as possible, so we set $V = 1000$. Our cost equation is now:

$$C = 120y + \frac{4000}{y} + 200$$

Taking the derivative and setting it equal to zero,

$$\frac{dC}{dy} = 120 - \frac{4000}{y^2} = 0,$$

we find that the cost function has a positive critical point at $y = \sqrt{\frac{100}{3}} \approx 5.774$ feet. Since the second derivative of the cost function, $\frac{d^2C}{dy^2} = \frac{24000}{y^3}$, is positive for positive values of y, we know that the function is concave up for all $y > 0$ and this value of y is a minimum for the cost function. Solving for x, we find $x \approx 17.321$ feet.

Thus, the glass side is the small side, and the dimensions and cost are:

Dimensions: 5.774 x 17.321 x 10 feet

Minimum Cost: ~1585.74 dollars

[2]See `http://www.cephbase.utmb.edu/Tcp/pdf/anderson-wood.pdf`. (They really DO escape....)