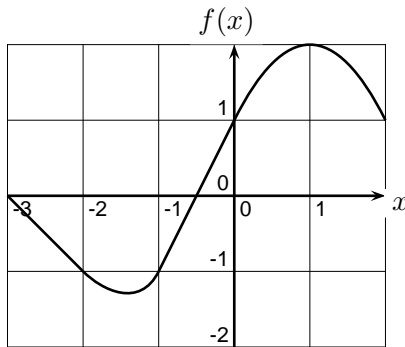


6. (12 points) The graph of a function f is shown below, together with a table of values for its derivative f' . Let $g(x) = f(f(x))$.



x	$f'(x)$
-3	-1
-2	-1
-1	2
0	2
1	0
2	-2

- (a) (2 points) Find $g(-2)$

$$g(-2) = f(f(-2)) = f(-1) = -1.$$

- (b) (3 points) Find $g'(-2)$

$$\text{By the chain rule, } g'(-2) = f'(f(-2)) \cdot f'(-2) = f'(-1) \cdot (-1) = (2)(-1) = -2.$$

- (c) (3 points) Write an expression for $g''(x)$ in terms of f and its derivatives.

Again by the chain rule, $g'(x) = f'(f(x)) \cdot f'(x)$, so

$$g''(x) = f'(f(x)) \cdot f''(x) + f''(f(x)) \cdot f'(x) \cdot f'(x).$$

- (d) (4 points) Suppose $f''(-1) = 2$. What is $g''(-1)$?

Using the answer above,

$$\begin{aligned} g''(-1) &= f'(f(-1)) \cdot f''(-1) + f''(f(-1)) \cdot f'(-1) \cdot f'(-1) \\ &= (2) \cdot (2) + (2) \cdot (2)^2 \\ &= 4 + 8 = 12. \end{aligned}$$