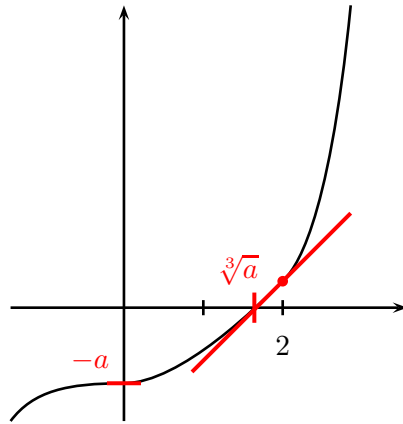


8. Let  $f(x) = x^3 - a$ , for  $a > 1$  a constant. The graph of  $f$  is shown below.

(a) (2 points) Find and label the numbers  $-a$  and  $\sqrt[3]{a}$  on the axes below.



(b) (4 points) Find the equation for  $L(x)$ , the linear approximation to  $f$  near  $x = 2$ . Your equation will contain the constant  $a$ . Sketch the graph of  $L(x)$  on the axes above.

In general, the equation for  $L$  is  $L(x) = f'(2)(x - 2) + f(2)$ . Since  $f(2) = 2^3 - a = 8 - a$ , and  $f'(2) = 3(2)^2 = 12$ , we get

$$L(x) = 12(x - 2) + 8 - a = 12x - 16 - a.$$

(c) (4 points) Use the function  $L$  to approximate  $2.01^3$ .

Since  $f(x) = x^3 - a$ , we see  $x^3 = f(x) + a$ . Using  $L$  to approximate  $f$ , we get  $x^3 \approx L(x) + a = 12x - 16$  for  $x$  near 2. So

$$2.01^3 \approx 12(2.01) - 16 = 8.12.$$