2. (12 points) Suppose a is a positive (non-zero) constant, and consider the function

$$f(x) = \frac{1}{3}x^3 - 4a^2x.$$

Determine all maxima and minima of f in the interval $[-3a, 5a]$. For each, specify whether it is global or local.

We need to check values of f at the endpoints ($x = -3a$ and $x = 5a$) and wherever $f'(x) = 0$ or is undefined. Since $f'(x) = x^2 - 4a^2$, $f'(x)$ is defined for all x and $f'(x) = 0$ at $x = \pm 2a$. So, we have critical points $x = -2a, 2a$. We check all points individually to determine which are minima and which are maxima. We can use the second derivative, $f''(x) = 2x$ to help with the check.

- $\underline{x = -3a}$:
  $f'(-3a) = (-3a)^2 - 4a^2 = 5a^2 > 0$, so the function is increasing there, and this endpoint must be a (local) minimum. Since $f(-3a) = 3a^3$, we have the point $(-3a, 3a^3)$.
- $\underline{x = 5a}$:
  $f'(5a) = (5a)^2 - 4a^2 = 21a^2 > 0$, so this endpoint must be a (local) maximum. Since $f(5a) = \frac{65}{3}a^3$, we have the point $(5a, \frac{65}{3}a^3)$.
- $\underline{x = -2a}$:
  $f''(-2a) = -4a < 0$, so this must be a (local) maximum. Since $f(-2a) = \frac{16}{3}a^3$, we see that this occurs at the point $(-2a, \frac{16}{3}a^3)$.
- $\underline{x = 2a}$:
  $f''(2a) = 4a > 0$, so this must be a (local) minimum. Since $f(2a) = -\frac{16}{3}a^3$, we see that this minimum occurs at the point $(2a, -\frac{16}{3}a^3)$.

Comparing the y-values of the minima at $x = -3a, 2a$, we find that the global minimum occurs at $x = 2a$. Similarly, comparing the y-values of the maxima at $x = -2a, 5a$, we find the global maximum at the endpoint $x = 5a$.
Summing up:

- There's a local minimum at $(-3a, 3a^3)$.
- There's a local maximum at $(-2a, \frac{16}{3}a^3)$.
- There's a local and global minimum at $(2a, -\frac{16}{3}a^3)$.
- There's a local and global maximum at $(5a, \frac{65}{3}a^3)$.