3. Table 1 below displays some values of an invertible, differentiable function $f(x)$, while Figure 2 depicts the graph of the function $g(x)$. Set $h(x) = f\big(g(x)\big)$ and $j(x) = \dfrac{f(x)}{g(x)}$.

Table 1

x	1	2	3	4	5
$f(x)$	-5	-2	2	4	7
$f'(x)$	5	6	2	3	3
$f''(x)$	1	-1	-3	-2	0

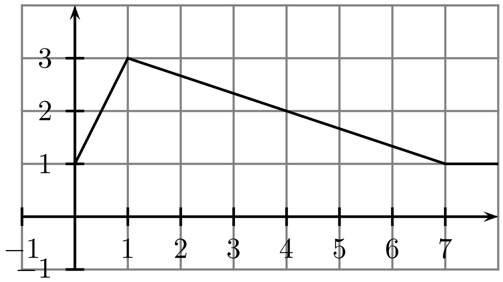

Figure 2: Graph of $g(x)$

Evaluate each of the following. **To receive partial credit you must show your work!**

(a) (4 points) $\left(f^{-1}\right)'(2)$

$$\left(f^{-1}\right)'(2) = \frac{1}{f'\big(f^{-1}(2)\big)} = \frac{1}{f'(3)} = \frac{1}{2}$$

(b) (4 points) $h'(4)$

By the chain rule, $h'(x) = f'\big(g(x)\big) \cdot g'(x)$. Therefore,

$$h'(4) = f'\big(g(4)\big) \cdot g'(4) = f'(2) \cdot \frac{-1}{3} = -2$$

(c) (4 points) $h''(4)$ [*Hint: you may want to use your work from part (b).*]

To find h'', we differentiate the formula we obtained for $h'(x)$ in part (b). Using the product rule and the chain rule, we find

$$\begin{aligned} h''(x) &= \Big(f'\big(g(x)\big)\Big)' \cdot g'(x) + f'\big(g(x)\big) \cdot g''(x) \\ &= f''\big(g(x)\big) \cdot g'(x) \cdot g'(x) + f'\big(g(x)\big) \cdot g''(x) \\ &= f''\big(g(x)\big) \cdot \big(g'(x)\big)^2 + f'\big(g(x)\big) \cdot g''(x) \end{aligned}$$

Therefore, $h''(4) = f''(2) \cdot \frac{1}{9} + f'(2) \cdot 0 = -\frac{1}{9} \approx -0.111$.

(d) (4 points) $j'(4)$

By the quotient rule,

$$j'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g(x)^2}.$$

Therefore,

$$j'(4) = \frac{g(4) \cdot f'(4) - f(4) \cdot g'(4)}{g(4)^2} = \frac{2 \cdot 3 + 4 \cdot \frac{1}{3}}{4} = \frac{11}{6} \approx 1.833.$$