5. Your friend starts a small company which sells awesome t-shirts for \$10 apiece. The table below shows the cost of making different numbers of shirts:

q (number of shirts made)	5	10	15	20	25	30	35	40	45	50
$C(q)$ (cost, in \$)	100	130	150	168	184	196	206	218	236	256

(a) (2 points) Write an expression for the revenue function $R(q)$.

$R(q) = 10q$, measured in dollars.

(b) (4 points) How many shirts should your friend aim to sell, if her goal is to maximize profit? Explain.

The profit function $\pi(q) = R(q) - C(q)$. The critical points of this function are those q which make $\pi'(q) = 0$, as well as the endpoints. We check these.

From above, we know that $\pi'(q) = R'(q) - C'(q) = 10 - C'(q)$. In the table above, the largest difference between consecutive values of $C(q)$ is 30 (which is $C(10) - C(5)$), which means the largest value $C'(q)$ takes on the interval is 6. Therefore, as far as we can glean from the information given in the table, we should expect that $\pi'(q)$ is positive everywhere in the interval $0 \leq q \leq 50$. So, the maximum should be at one of the endpoints.

When $q = 5$ (i.e. 5 shirts are sold), your friend loses money, since $\pi(5) = -50$. At the other extreme, if your friend sells 50 awesome t-shirts, she will make a tidy profit of $\pi(50) = 244$ dollars. Thus, she should aim to sell 50 shirts (and probably more, if that's a possibility).

6. (6 points) The radius of a spherical balloon is increasing by 3 cm per second. At what rate is air being blown into the balloon at the moment when the radius is 9 cm? Make sure you include units! [*Hint: the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.*]

If $V(t)$ denotes the volume of the balloon at some time t, the rate at which air is being blown into it is $\frac{dV}{dt}$. By chain rule, since the volume of the balloon is $V = \frac{4}{3}\pi r^3$, we have

$$\begin{aligned} \frac{dV}{dt} &= \frac{dV}{dr} \cdot \frac{dr}{dt} \\ &= (4\pi r^2) \cdot \frac{dr}{dt} \end{aligned}$$

We are given in the problem statement that $\frac{dr}{dt} = 3$, whence

$$\frac{dV}{dt} = 12\pi r^2.$$

Thus when $r = 9$,

$$\frac{dV}{dt} = 972\pi \approx 3053.6 \text{ cm}^3/\text{s}.$$