7. You decide to take a weekend off and drive down to Chicago. The graph below represents your distance \( S \) from Ann Arbor, measured in miles, \( t \) hours after you set out.

Let \( A(t) \) be the slope of the line connecting the origin \((0, 0)\) to the point \((t, S(t))\).

(a) (3 points) What does \( A(t) \) represent in everyday language?

\[ A(t) = \frac{S(t)}{t} \]
represents your average velocity, in miles per hour, during the first \( t \) hours of the trip.

(b) (3 points) Estimate the time \( t \) at which \( A(t) \) is maximized. Write a one sentence explanation and use the graph above to justify your estimate.

\( A(t) \) is maximized when the line connecting the origin to the curve is steepest. By inspecting the graph, it is clear that this happens at around \( t = 3 \) hours, at which point the line is tangent to the curve. (This line is indicated in the figure by the dashed line.)

(c) (4 points) Use calculus to explain why \( A(t) \) has a critical point when the line connecting the origin to the point \((t, S(t))\) is tangent to the curve \( S(t) \).

By applying the quotient rule, we find that

\[ A'(t) = \frac{t \cdot S'(t) - S(t)}{t^2}. \]

At any non-zero value of \( t \) for which the line through the origin to \((t, S(t))\) is tangent to the curve, we must have \( S'(t) \) (the slope of the tangent line) equal to \( S(t)/t \) (the slope of the line from the origin to \((t, S(t))\)); thus, for any such \( t \), \( S(t) = t \cdot S'(t) \). Thus, for any such \( t \), \( A'(t) = 0 \), i.e. \( A(t) \) has a critical point there.