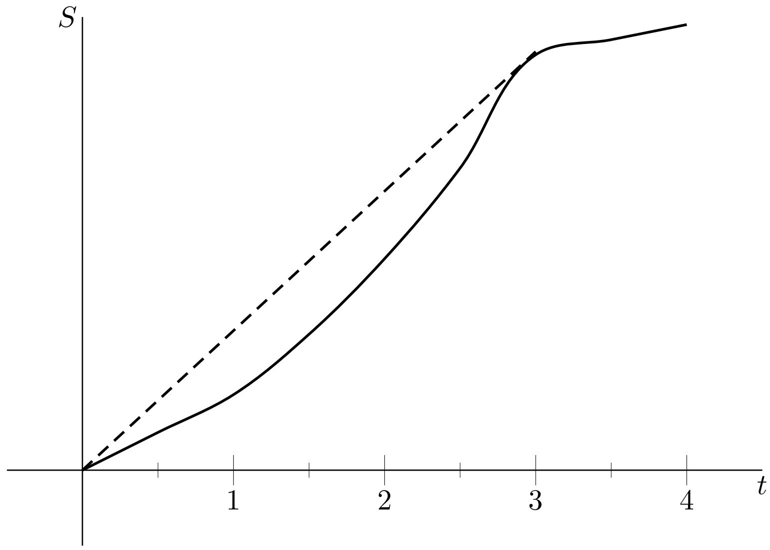
7. You decide to take a weekend off and drive down to Chicago. The graph below represents your distance $S$ from Ann Arbor, measured in miles, $t$ hours after you set out.




Let $A(t)$ be the slope of the line connecting the origin $(0,0)$ to the point $\big(t, S(t)\big)$.

(a) (3 points) What does $A(t)$ represent in everyday language?

> $A(t) = \dfrac{S(t)}{t}$ represents your average velocity, in miles per hour, during the first $t$ hours of the trip.

(b) (3 points) Estimate the time $t$ at which $A(t)$ is maximized. Write a one sentence explanation and use the graph above to justify your estimate.

> $A(t)$ is maximized when the line connecting the origin to the curve is steepest. By inspecting the graph, it is clear that this happens at around $t = 3$ hours, at which point the line is tangent to the curve. (This line is indicated in the figure by the dashed line.)

(c) (4 points) Use calculus to explain why $A(t)$ has a critical point when the line connecting the origin to the point $\big(t, S(t)\big)$ is tangent to the curve $S(t)$.

> By applying the quotient rule, we find that
>
> $$A'(t) = \frac{t \cdot S'(t) - S(t)}{t^2}.$$
>
> At any non-zero value of $t$ for which the line through the origin to $\big(t, S(t)\big)$ is tangent to the curve, we must have $S'(t)$ (the slope of the tangent line) equal to $S(t)/t$ (the slope of the line from the origin to $\big(t, S(t)\big)$); thus, for any such $t$, $S(t) = t \cdot S'(t)$. Thus, for any such $t$, $A'(t) = 0$, i.e. $A(t)$ has a critical point there.