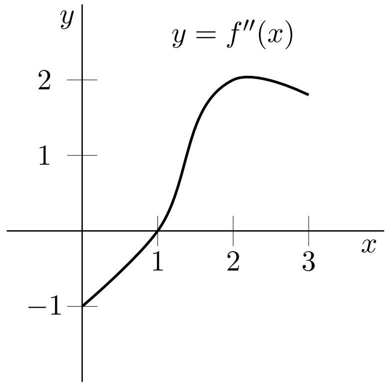
8. The figure below shows the graph of the *second* derivative of f, on the interval $[0, 3]$.



Assume that $f'(1) = 1$ and $f(1) = 0$.

(a) (5 points) Can $f'(x) = 0.5$ for some x in $[0, 3]$? Why or why not?

$f'(x)$ is decreasing on the interval $[0, 1]$ (since $f''(x) \leq 0$), and increasing on the interval $[1, 3]$ (since $f''(x) \geq 0$). Thus, on the whole interval $[0, 3]$, $f'(x)$ has a minimum at $x = 1$. Since $f'(1) = 1$, we deduce that $f'(x) \geq 1$ for all x in the interval $[0, 3]$. In particular, $f'(x)$ cannot equal 0.5 in the interval.

(b) (5 points) Explain why f has a global maximum at $x = 3$.

From above, $f'(x) \geq 1$ for all x in the interval $[0, 3]$. In particular, $f'(x)$ is always positive, so f is everywhere increasing on this interval. This means f attains its global maximum at the rightmost endpoint of the interval, namely, at $x = 3$.