2. [12 points]

Use the graph of the function $f$ and the table of values for the function $g$ to answer the questions below.


| x | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~g}(\mathrm{x})$ | 0 | 4 | 0 | -18 | -56 | -120 |
| $\mathrm{~g}^{\prime}(\mathrm{x})$ | 6 | 1 | -10 | -27 | -50 | -79 |
| $\mathrm{~g}^{\prime \prime}(\mathrm{x})$ | -2 | -8 | -14 | -20 | -26 | -32 |

a. [6 points] Let $h(x)=\frac{g(x)}{f(2 x+3)}$. Find $h^{\prime}(1)$ or explain why it does not exist.

Solution: Using the quotient rule and the chain rule, we get

$$
\begin{aligned}
h^{\prime}(x) & =\frac{g^{\prime}(x) f(2 x+3)-g(x) f^{\prime}(2 x+3) \cdot 2}{(f(2 x+3))^{2}} \\
h^{\prime}(1) & =\frac{g^{\prime}(1) f(5)-g(1) f^{\prime}(5) \cdot 2}{(f(5))^{2}} \\
& =\frac{6 \cdot 2.5-0 \cdot 0.75 \cdot 2}{(2.5)^{2}} \\
& =\frac{6}{2.5}=\frac{12}{5}=2.4
\end{aligned}
$$

b. [6 points] Let $k(x)=g(g(x))$. Determine whether $k$ is increasing or decreasing at $x=2$.

Solution: Using the chain rule, we get

$$
\begin{aligned}
k^{\prime}(x) & =g^{\prime}(g(x)) \cdot g^{\prime}(x) \\
k^{\prime}(2) & =g^{\prime}(g(2)) \cdot g^{\prime}(2) \\
& =g^{\prime}(4) \cdot g^{\prime}(2) \\
& =(-27) \cdot 1=-27
\end{aligned}
$$

Since $k^{\prime}(2)<0$, we know that $k(x)$ is decreasing at $x=2$.

