3. [10 points] Find the $x$- and $y$-coordinates of all local minima, local maxima, and inflection points of the function $f(x)$ defined below. Your answers may involve the positive constant $B$. You must clearly mark your answers and provide justification to receive credit.

$$f(x) = e^{-18x^2 + B}$$

Solution: We will need both $f'(x)$ (chain rule) and $f''(x)$ (product rule and chain rule).

$$f'(x) = e^{-18x^2 + B}(-36x) = -36xe^{-18x^2 + B}$$

$$f''(x) = -36e^{-18x^2 + B} + (-36x)e^{-18x^2 + B}(-36x)$$

$$= -36e^{-18x^2 + B}(1 - 36x^2)$$

First we will find the critical points. We know $f'$ is never undefined, and $f'(x) = 0$ only when $x = 0$, since $e^{-18x^2 + B}$ is always positive. The $y$-value for $x = 0$ is $e^{-18\cdot0^2 + B} = e^B$. Thus, $(0,e^B)$ is the only critical point. Since $f''(0) = -36e^B(1) = -36e^B$ is negative, we know this point is a local maximum.

LOCAL MAXIMUM AT $(0,e^B)$.

Next we will find potential inflection points. We know $f''$ is never undefined, and $f''(x) = 0$ when $1 - 36x^2 = 0$, since $e^{-18x^2 + B}$ is always positive. Solving, we find that $f''(x) = 0$ when $x = \pm \frac{1}{6}$. Both of these points have a $y$-value of $e^{-18(\frac{1}{6})^2 + B} = e^{-\frac{1}{2} + B}$. We need to test $f''$ near these $x$-values to check whether we actually have inflection points.

When $x < -\frac{1}{6}$, $f''(x) > 0$.

When $-\frac{1}{6} < x < \frac{1}{6}$, $f''(x) < 0$.

When $x > \frac{1}{6}$, $f''(x) > 0$.

Since $f''$ changes sign at both of these points, $f$ changes concavity at both points, so both are inflection points.

INFLECTION POINTS AT $(-\frac{1}{6},e^{-\frac{1}{2}+B})$ and $(\frac{1}{6},e^{-\frac{1}{2}+B})$. 