3. [10 points]
Find the x- and y-coordinates of all local minima, local maxima, and inflection points of the function $f(x)$ defined below. Your answers may involve the positive constant B. You must clearly mark your answers and provide justification to receive credit.

$$f(x) = e^{-18x^2+B}$$

Solution: We will need both $f'(x)$ (chain rule) and $f''(x)$ (product rule and chain rule).

$$\begin{aligned} f'(x) &= e^{-18x^2+B}(-36x) = -36xe^{-18x^2+B} \\ f''(x) &= -36e^{-18x^2+B} + (-36x)e^{-18x^2+B}(-36x) \\ &= -36e^{-18x^2+B}(1-36x^2) \end{aligned}$$

First we will find the critical points. We know f' is never undefined, and $f'(x) = 0$ only when $x = 0$, since e^{-18x^2+B} is always positive. The y-value for $x = 0$ is $e^{-18\cdot 0^2+B} = e^B$ Thus, $(0, e^B)$ is the only critical point. Since $f''(0) = -36e^B(1) = -36e^B$ is negative, we know this point is a local maximum.

LOCAL MAXIMUM AT $(0, e^B)$.

Next we will find potential inflection points. We know f'' is never undefined, and $f''(x) = 0$ when $1 - 36x^2 = 0$, since e^{-18x^2+B} is always positive. Solving, we find that $f''(x) = 0$ when $x = \pm\frac{1}{6}$. Both of these points have a y-value of $e^{-18(\frac{1}{6})^2+B} = e^{-\frac{1}{2}+B}$. We need to test f'' near these x-values to check whether we actually have inflection points.
When $x < -\frac{1}{6}$, $f''(x) > 0$.
When $-\frac{1}{6} < x < \frac{1}{6}$, $f''(x) < 0$.
When $x > \frac{1}{6}$, $f''(x) > 0$.
Since f'' changes sign at both of these points, f changes concavity at both points, so both are inflection points.

INFLECTION POINTS AT $(-\frac{1}{6}, e^{-\frac{1}{2}+B})$ and $(\frac{1}{6}, e^{-\frac{1}{2}+B})$.