7. [16 points] Janet is an artist who produces and sells prints of her artwork. If Janet sells her prints for \$17 each, then she will sell 340 prints. Janet is considering whether she should change the price. She takes a survey and concludes that for each price increase of 75 cents, she will sell 10 fewer prints.

a. [4 points] Find a formula for Janet's revenue, $R(x)$, in terms of x, the number of 75 cent price increases.

Solution: Since revenue is price times quantity, we have

$$R(x) = (17 + 0.75x)(340 - 10x).$$

b. [4 points] Janet plans to produce exactly the number of prints that her survey predicts she will sell. Her costs include \$2 per print, along with \$500 in fixed costs. Find a formula for $C(x)$, Janet's total costs, in terms of x, the number of 75 cent price increases.

Solution: Fixed costs = \$500, cost per print = \$2 per print, and number of prints = $340 - 10x$, so

$$C(x) = 500 + 2(340 - 10x)$$

c. [8 points] Use the methods of calculus to determine what price Janet should set for her prints if she wants to maximize her profit.

Solution: Let $\pi(x) = \text{profit} = R(x) - C(x)$:

$$\pi(x) = (340 - 10x)(17 + 0.75x) - [500 + 2(340 - 10x)]$$

So,

$$\begin{aligned} \pi'(x) &= (-10)(17 + 0.75x) + (340 - 10x)(0.75) - 2(-10) \\ &= -170 - 7.5x + 255 - 7.5x + 20 \\ &= -15x + 105. \end{aligned}$$

Note that $\pi'(x)$ is defined for all x, and $\pi'(x) = 0$ if $15x = 105$ or $x = 7$. Thus, we have one critical point at $x = 7$.

We must test the critical point. Using the second derivative test, we have $\pi''(x) = -15$ which is negative for all x, so $x = 7$ gives a local maximum. Since $\pi(x)$ is continuous, and we have only one critical point, $x = 7$ is a global max. Thus, $x = 7$ maximizes the profit.

The price that Janet should charge is $17 + 0.75(7) = 17 + 5.25 = \$22.25$.