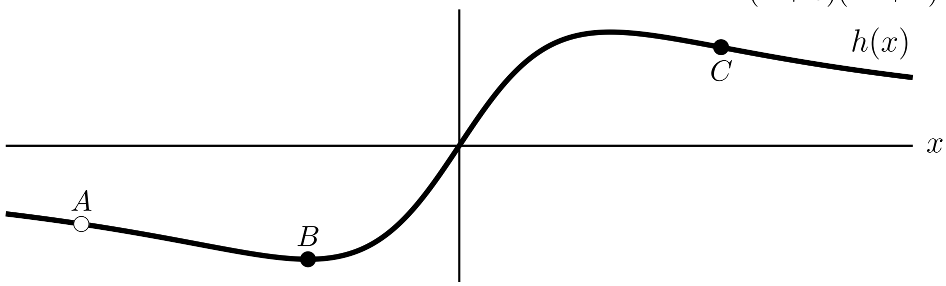
8. [13 points] Below, there is a graph of the function $h(x) = \dfrac{2x^2 + 10x}{(x+5)(x^2+4)}$.



a. [3 points] The point A is a hole in the graph of h. Find the x- and y-coordinates of A.

Solution: Simplifying $h(x)$, we have $h(x) = \dfrac{2x(x+5)}{(x+5)(x^2+4)}$. Since the factor $(x+5)$ cancels, the hole occurs when $x = -5$. We look at the limit as x approaches -5 on the cancelled form to get the y-coordinate:

$$\lim_{x \to -5} h(x) = \lim_{x \to -5} \frac{2x}{x^2+4} = \frac{-10}{29},$$

Thus, $A = (-5, \frac{-10}{29})$.

b. [5 points] The point B is a local minimum of h. Find the x- and y-coordinates of B.

Solution: Using the quotient rule on the simplified form of h, we have $h'(x) = \dfrac{4 - x^2}{(x^2+4)^2}$. This is never undefined, and it is equal to zero when $4 - x^2 = 0$ or $x = \pm 2$. From the graph, we can see that the local minimum occurs at $x = -2$. The y-coordinate here is $y = \frac{-4}{8} = -\frac{1}{2}$, so $B = (-2, -\frac{1}{2})$.

c. [5 points] The point C is an inflection point of h. Find the x- and y-coordinates of C.

Solution: We use the quotient rule again to find $h''(x) = \dfrac{2x^3 - 24x}{(x^2+4)^3} = \dfrac{2x(x^2-12)}{(x^2+4)^3}$. This is never undefined, and it is zero when $2x(x^2 - 12) = 0$, i.e. when $x = 0, \pm 2\sqrt{3}$. From the graph, we see that our x-coordinate must be $+2\sqrt{3}$, and then $y = \frac{4\sqrt{3}}{16} = \frac{\sqrt{3}}{4}$, so $C = (2\sqrt{3}, \frac{\sqrt{3}}{4})$.