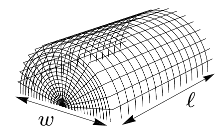
1. [15 points] A hoophouse is an unheated greenhouse used to grow certain types of vegetables during the harsh Michigan winter. A typical hoophouse has a semi-cylindrical roof with a semi-circular wall on each end (see figure to the right). The growing area of the hoophouse is the rectangle of length ℓ and width w (each measured in feet) which is covered by the hoophouse. The cost of the semi-circular walls is \$0.50 per square foot and the cost of the roof, which varies with the side length ℓ, is $\$1 + 0.001\ell$ per square foot.



a. [4 points] Write an equation for the cost of a hoophouse in terms of ℓ and w. *(Hint: The surface area of a cylinder of height ℓ and radius r, not including the circles on each end, is $A = 2\pi r\ell$.)*

Solution: The roof has area $\pi r\ell = \frac{\pi}{2}w\ell$. The walls have area $\pi r^2 = \frac{\pi}{4}w^2$. This means the cost is

$$C = 0.50 \cdot \frac{\pi}{4}w^2 + (1 + 0.001\ell)\frac{\pi}{2}w\ell = \frac{\pi}{8}w^2 + \frac{\pi}{2}(1 + 0.001\ell)w\ell.$$

b. [11 points] Find the dimensions of the least expensive hoophouse with 8000 square feet of growing area.

Solution: The Area of the hoophouse is $8000 = w\ell$. Using this expression, we can eliminate ℓ in our cost equation.

$$C = \frac{\pi}{8}w^2 + \frac{\pi}{2}(1 + 0.001\ell)w\ell = \frac{\pi}{8}w^2 + \frac{\pi}{2}(1 + 0.001(8000/w))8000.$$
$$= 4000\pi + \frac{\pi}{8}w^2 + 32000\pi w^{-1}.$$

Now we compute $C' = \frac{\pi}{4}w - 32000\pi w^{-2}$. Solving for w gives us a critical point at $w =$ 50.397ft. To see what type of critical point we have, we compute $C'' = \frac{\pi}{4} + 64000\pi w^{-3}$. For $w > 0$ $C'' > 0$ which means our critical point is a local minimum by the second derivative test. Since it is the only critical point of the function, it must be a global minimum as well. When $w = 50.397$, $\ell = 158.74$, so the least expensive hoophouse with 8000 square feet of growing area is 50.397 x 158.74 ft.