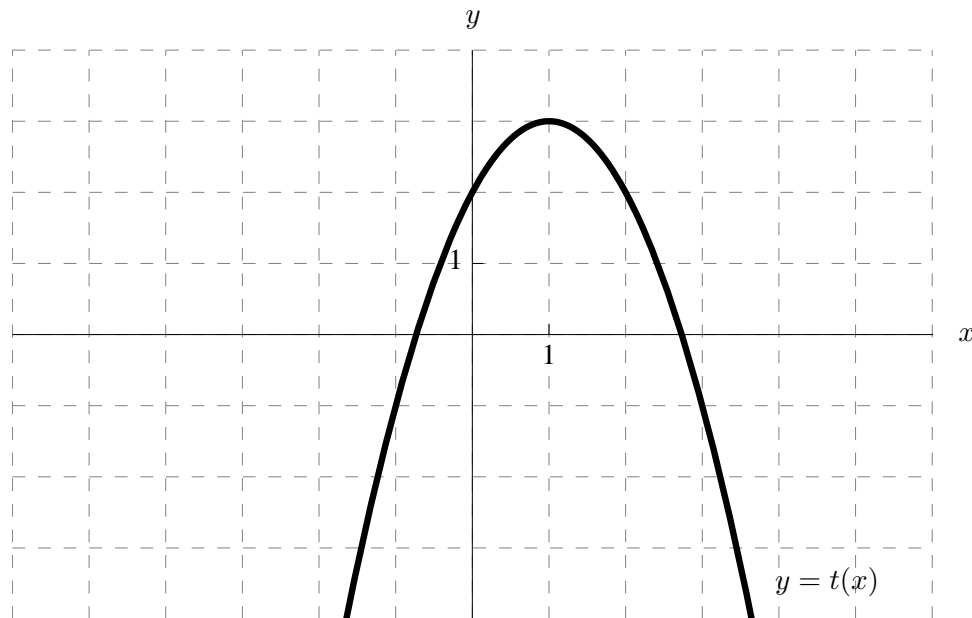


2. [16 points]

Graphed below is a function $t(x)$. Define $p(x) = x^2t(x)$, $q(x) = t(\sin(x))$, $r(x) = \frac{t(x)}{3x+1}$, and $s(x) = t(t(x))$. For this problem, do not assume $t(x)$ is quadratic.



Carefully estimate the following quantities.

a. [4 points] $p'(-1)$

Solution: By the product rule, $p'(x) = 2xt(x) + x^2t'(x)$. Estimating using the graph, we have

$$p'(-1) = 2(-1)t(-1) + (-1)^2t'(-1) = (-2)(-1) + 4 = 6.$$

b. [4 points] $q'(0)$

Solution: By the chain rule, $q'(x) = t'(\sin x) \cos x$. Estimating using the graph, we have

$$q'(0) = t'(\sin 0) \cos 0 = t'(0) = 2.$$

c. [4 points] $r'(3)$

Solution: By the quotient rule, $r'(x) = \frac{(3x+1)t'(x) - 3t(x)}{(3x+1)^2}$. Estimating using the graph, we have

$$r'(3) = \frac{(3(3) + 1)t'(3) - 3t(3)}{(3(3) + 1)^2} = \frac{-40 - 3(-1)}{100} = -\frac{37}{100}$$

d. [4 points] $s'(0)$

Solution: By the chain rule, $s'(x) = t'(t(x))t'(x)$. Estimating using the graph, we have

$$s'(0) = t'(t(0))t'(0) = t'(2) \cdot 2 = (-2)(2) = -4.$$