3. [12 points] Representative values of the derivative of a function $f(x)$ are shown in the table below. Assume $f'(x)$ is a continuous function and that the values in the table are representative of the behavior of $f'(x)$.

x	0	0.5	1	1.5	2	2.5	3
$f'(x)$	1	0.3	0	-0.1	-0.15	-0.12	-0.10

a. [6 points] Estimate the location of the global maximum and minimum of $f(x)$ on the closed interval $[0, 3]$. Justify your answers based on the data in the table.

Solution: We note that $f'(x) > 0$ for $x < 1$ and $f'(x) < 0$ for $x > 1$. Thus $f(x)$ has a local maximum at $x = 1$. Further, because there is only one change of sign in the derivative, we know that this is the global maximum. The global minimum will occur at one of the endpoints. It is not easy to tell at which endpoint this occurs, but because the negative slopes are of smaller magnitude (for $x > 1$) than the positive slopes (for $x < 1$), we expect that the global minimum occurs at $x = 0$.

b. [6 points] Can you tell from these data if $f(x)$ has any inflection points? If so, estimate the location of any inflection points and indicate how you know their locations. If not, explain why not.

Solution: We know that an inflection point occurs when $f'(x)$ goes from increasing to decreasing or vice versa. We can see from these data that $f'(x)$ is decreasing until sometime between $x = 2$ and $x = 2.5$, and increasing thereafter. Thus there is an inflection point at an x somewhere in $(2, 2.5)$.