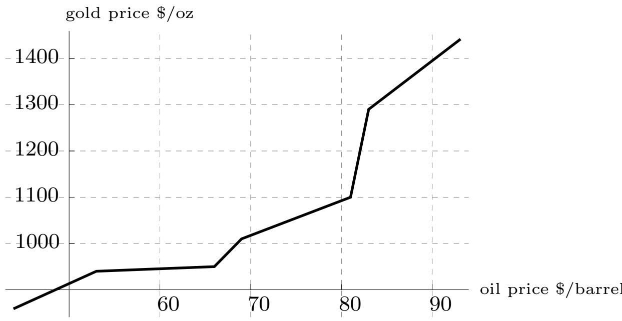
5. [15 points] The graph to the right shows a function $G(b)$ that approximates the price of an ounce of gold (in dollars) as a function of the cost of a barrel of oil for data between 2009 and 2011.[1]



a. [3 points] Estimate $G'(70)$.

Solution: From the graph, it appears that between $b = 70$ and $b = 80$, G increases by about 70 as b increases about 10. Thus we estimate that $G'(70) \approx 7$ $/oz per $/barrel.

b. [5 points] Recall that G^{-1} is defined to be a function such that $G^{-1}(G(b)) = b$ (or such that $G(G^{-1}(y)) = y$, where y is the price of an ounce of gold). Derive, using the chain rule, a formula for $(G^{-1})'$ in terms of G'.

Solution: We know that $G^{-1}(G(b)) = b$. Thus $\frac{d}{db}G^{-1}(G(b)) = 1$. Differentiating the left-hand side of this using the chain rule, we have $\frac{d}{db}G^{-1}(G(b)) = (G^{-1})'(G(b)) \cdot G'(b) = 1$. Thus $(G^{-1})'(G(b)) = 1/G'(b)$.
Alternately, if we start with $G(G^{-1}(y)) = y$, we have $\frac{d}{dy}G(G^{-1}(y)) = 1$. Applying the chain rule to the left-hand side, we have $G'(G^{-1}(y)) \cdot (G^{-1})'(y) = 1$, so that $(G^{-1})'(y) = 1/G'(G^{-1}(y))$. (Obviously, with $y = G(b)$, this is the same as the previous expression.)

c. [4 points] Using parts (a) and (b), estimate $(G^{-1})'(G(70))$.

Solution: Using part (b), we have $(G^{-1})'(G(70)) = 1/G'(70) = 1/7$ $/barrel per $/oz.

d. [3 points] Explain the practical meaning of your result in (c).

Solution: $(G^{-1})'(G(70)) = 0.14$ indicates that when the price of oil is 70 $/barrel, the price of a barrel of oil goes up by about $0.14 if the price of gold goes up by $1.

[1] Gold prices from <http://www.goldprice.org/>; oil from <http://en.wikipedia.org/wiki/Price_of_petroleum>.