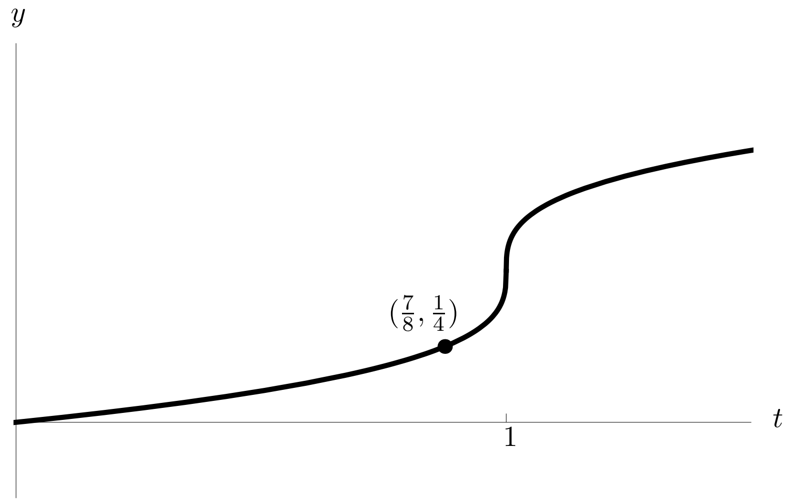
6. [15 points] Given below is the graph of a function $h(t)$. Suppose $j(t)$ is the local linearization of $h(t)$ at $t = \frac{7}{8}$.



a. [5 points] Given that $h'(\frac{7}{8}) = \frac{2}{3}$, find an expression for $j(t)$.

Solution: The local linearization is the tangent line to the curve. We know this line has slope $h'(\frac{7}{8}) = \frac{2}{3}$ and it goes through the point $(\frac{7}{8}, \frac{1}{4})$, so it has equation

$$y - \frac{1}{4} = \frac{2}{3}(t - \frac{7}{8})$$

using point slope form. Solving for y we have $y = \frac{2}{3}t - \frac{1}{3}$. So $j(t) = \frac{2}{3}t - \frac{1}{3}$. stuff

b. [4 points] Use your answer from (a) to approximate $h(1)$.

Solution: Since $j(t)$ approximates $h(t)$ for t-values near $\frac{7}{8}$, we have

$$h(1) \approx j(1) = \frac{2}{3}(1) - \frac{1}{3} = \frac{1}{3}.$$

c. [3 points] Is the approximation from (b) an over- or under-estimate? Explain.

Solution: The approximation in (b) is an underestimate. The function $h(t)$ is concave up at $t = 7/8$ which means the graph lies above the local linearization for t-values near $7/8$. Since we are using the local linearization to estimate the function value, our estimate will be less than the actual function value.

d. [3 points] Using $j(t)$ to estimate values of $h(t)$, will the estimate be more accurate at $t = 1$ or at $t = \frac{3}{4}$? Explain.

Solution: The estimate at $t = 3/4$ will be more accurate. This can be seen by drawing the tangent line and measuring the vertical distance between the estimated value and the function value at the t values $3/4$ and 1. The line is much closer to the function at $t = 3/4$ than it is at $t = 1$.