7. [12 points] On the axes below are graphed $f, f^{\prime}$, and $f^{\prime \prime}$. Determine which is which, and justify your response with a brief explanation.


Solution: Looking to the far right of the graph, curve I has a critical point where it has a slope of zero. At this x-coordinate neither of the other graphs has a root. This means the derivative of $\mathbf{I}$ is not in this figure, so $\mathbf{I}$ must be $f^{\prime \prime}$. Looking to the far left of the graph, II has a local maximum where its derivative is zero. Although III has a root near the same x-value, III changes sign from negative to positive at this point. By the first derivative test, III cannot be the derivative of II. Thus, by process of elimination, II must be $f^{\prime}$ and III must be $f$.

$$
\begin{aligned}
& f: \quad \text { III } \\
& f^{\prime}: \quad \text { II } \\
& f^{\prime \prime}: \quad \text { I }
\end{aligned}
$$

