8. [16 points] Below is the graph of the function

\[ f(x) = rx e^{-qx}, \]

where \( r \) and \( q \) are constants. Assume that both \( r \) and \( q \) are greater than 1. The function \( f(x) \) passes through the origin and has a local maximum at the point \( P = \left( \frac{1}{q}, \frac{r}{q} e^{-1} \right) \), as shown in the graph.

\[ \left( \frac{1}{q}, \frac{r}{q} e^{-1} \right) \]

\[ f(x) \]

\[ x \]

a. [4 points] Justify, using either the first-derivative test or second-derivative test, that the point \( P \) is a local maximum.

b. [2 points] What are the \( x \)-coordinates of the global maximum and minimum of \( f(x) \) on the domain \([0, 1]\)? (If \( f(x) \) does not have a global maximum on this domain, say “no global maximum”, and similarly if \( f(x) \) does not have a global minimum.)

c. [2 points] What are the \( x \)-coordinates of the global maximum and minimum of \( f(x) \) on the domain \(( -\infty, \infty )\)? (If \( f(x) \) does not have a global maximum on this domain, say “no global maximum”, and similarly if \( f(x) \) does not have a global minimum.)
8. (continued) For your convenience, the graph of $f(x)$ is repeated below.

![Graph of $f(x)$](image)

\[ \left( \frac{1}{q}, \frac{r}{q}e^{-1} \right) \]

**d.** [4 points] Suppose that $g(x)$ is a function with $g'(x) = f(x)$. Find $x$-values of all local maxima and minima of $g(x)$. Justify that each maximum you find is a maximum and each minimum is a minimum.

**e.** [4 points] If $g(x)$ is as in part (d), for which $x$-values does $g(x)$ have inflection points? Show that these $x$-values are indeed inflection points.