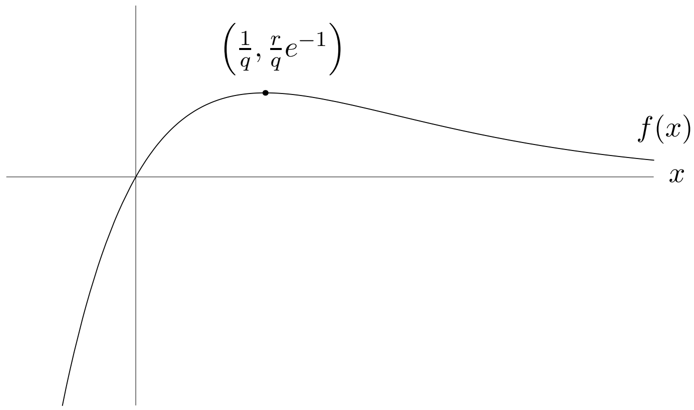
8. [16 points] Below is the graph of the function

$$f(x) = rxe^{-qx},$$

where r and q are constants. Assume that both r and q are greater than 1. The function $f(x)$ passes through the origin and has a local maximum at the point $P = \left(\frac{1}{q}, \frac{r}{q}e^{-1}\right)$, as shown in the graph.

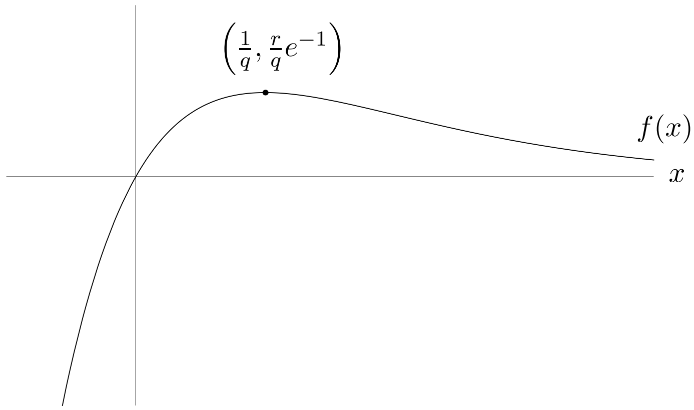


a. [4 points] Justify, using either the first-derivative test or second-derivative test, that the point P is a local maximum.

b. [2 points] What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain $[0, 1]$? (If $f(x)$ does not have a global maximum on this domain, say "no global maximum", and similarly if $f(x)$ does not have a global minimum.)

c. [2 points] What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain $(-\infty, \infty)$? (If $f(x)$ does not have a global maximum on this domain, say "no global maximum", and similarly if $f(x)$ does not have a global minimum.)

8. (continued) For your convenience, the graph of $f(x)$ is repeated below.



d. [4 points] Suppose that $g(x)$ is a function with $g'(x) = f(x)$. Find x-values of all local maxima and minima of $g(x)$. Justify that each maximum you find is a maximum and each minimum is a minimum.

e. [4 points] If $g(x)$ is as in part (d), for which x-values does $g(x)$ have inflection points? Show that these x-values are indeed inflection points.