2. [10 points] Lucy Lemon, the owner of a local lemonade stand, has observed that her lemonade sales are highly dependent on the temperature. Let $L(T)$ denote the number of cups of lemonade Lucy sells on a day whose average temperature is $T^{\circ}$ Fahrenheit. Below is a table of values for the derivative, $L^{\prime}(T)$. Assume that between each pair of consecutive $T$-values in the table, $L^{\prime}$ is either strictly increasing or strictly decreasing.

| $T$ | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L^{\prime}(T)$ | -5 | -4 | -2 | 2 | 5 | 6 | 3 | -2 | -3 |

a. [4 points] Approximate the values of $T$ at which $L(T)$ has a critical point, and classify each as a local maximum, local minimum, or neither. (No explanation is necessary.)

Solution: The sign of the derivative goes from negative to positive at approximately $T=62.5$, so by the first-derivative test, there is a local minimum at approximately that point. (Any $T$-value between 60 and 65 is an acceptable answer.) The sign of the derivative goes from positive to negative at approximately $T=82.5$, so there is a local maximum at approximately that point. (Any $T$-value between 80 and 85 is an acceptable answer.)
b. [6 points] Suppose on a day when the temperature is $T^{\circ}$ Fahrenheit, Lucy sells $R(T)$ dollars worth of lemonade. Assuming cups of lemonade sell for $\$ 1.50$ each, compute $R^{\prime}(80)$, and write a sentence expressing the meaning of $R^{\prime}(80)$ which would be understood by someone who knows no calculus.

Solution: The function $R(T)$ is given by the formula $R(T)=1.5 L(T)$, so

$$
R^{\prime}(80)=1.5 L^{\prime}(80)=4.5
$$

In practical terms, this means that Lucy earns about $\$ 4.50$ more on a day whose average temperature is $81^{\circ} \mathrm{F}$ than she does on a day whose average temperature is $80^{\circ} \mathrm{F}$.

