3. [12 points] The following questions relate to the implicit function

\[ y^2 + 4x = 4xy^2. \]

a. [4 points] Compute \( \frac{dy}{dx} \).

**Solution:** Differentiating the equation with respect to \( x \), we have

\[ 2y \frac{dy}{dx} + 4 = 4y^2 + 8xy \frac{dy}{dx}. \]

Gathering terms involving \( \frac{dy}{dx} \) to one side, the equation becomes

\[ 2y \frac{dy}{dx} - 8xy \frac{dy}{dx} = 4y^2 - 4, \]

which gives the solution

\[ \frac{dy}{dx} = \frac{4y^2 - 4}{2y - 8xy}. \]

b. [4 points] Find the equation for the tangent line to this curve at the point \((\frac{1}{3}, 2)\).

**Solution:** The slope is

\[ \frac{dy}{dx} \bigg|_{(\frac{1}{3}, 2)} = \frac{4 \cdot 2^2 - 4}{2 \cdot 2 - 8 \cdot \frac{1}{3} \cdot 2} = -9, \]

so by the point-slope formula, the equation is

\[ y = -9x + 5. \]

c. [4 points] Find the \( x \)- and \( y \)-coordinates of all points at which the tangent line to this curve is vertical.

**Solution:** The slope is undefined as these points, so we must have \( 2y - 8xy = 0 \). Factoring out a 2y we get

\[ 2y(1 - 4x) = 0 \]

which gives the solutions \( y = 0 \) or \( x = \frac{1}{4} \). Plugging into the equation for the implicit function, \( y = 0 \) gives the point \((0, 0)\). However, when we plug in \( x = \frac{1}{4} \), we get the equation \( y^2 + 1 = y^2 \), which has no solutions. Therefore, \((0, 0)\) is the only point at which the tangent line is vertical.